3D MHD Simulations of Turbulent Convection and Dynamo Action in Stars

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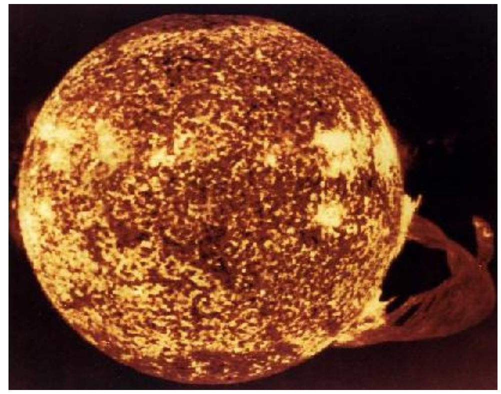
- Observational evidence and motivation
- 3-D MHD models of the solar convection zone
- Studying differential rotation & meridional circulation
- Studying elements of the of solar dynamo
- Role of rotation?
- 3-D MHD models of the core convection in A-type stars
- 3-D HD models of the Young Sun

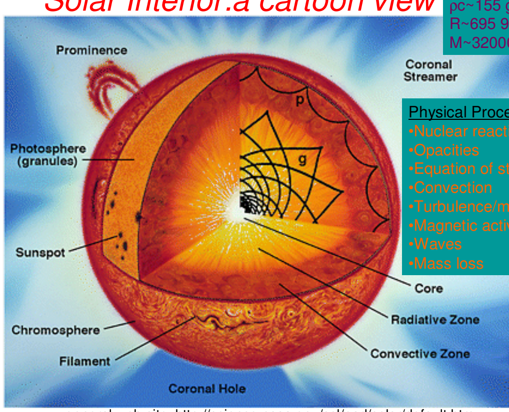
The Sun



SoHO data

Solar Interior:a cartoon view

Tc~15.5 10^6 K
ρc~155 g/cm3
R~695 990km
M~320000 M_{terre}

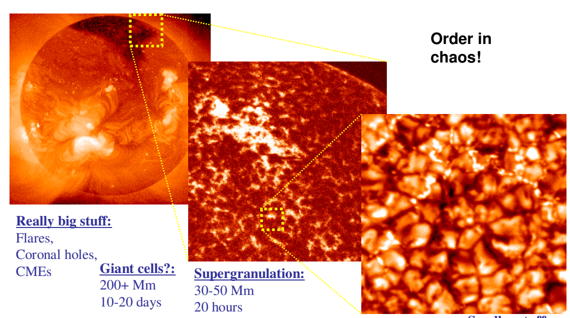


Physical Processes:
- Nuclear reaction
- Opacities
- Equation of state
- Convection
- Turbulence/mixing
- Magnetic activity
- Waves
- Mass loss

general web site: http://science.nasa.gov/ssl/pad/solar/default.htm

Solar Convection Scales

Order in chaos!



Really big stuff:
Flares,
Coronal holes,
CMEs

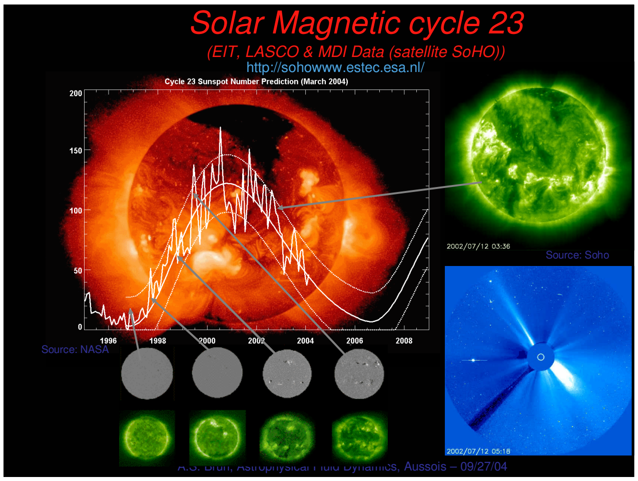
Giant cells?:
200+ Mm
10-20 days

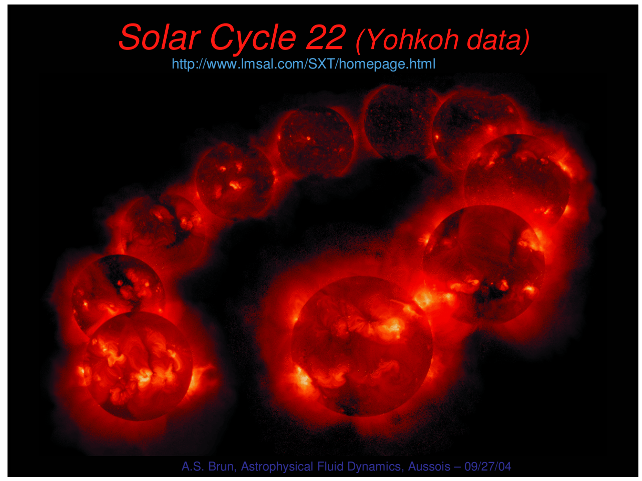
Supergranulation:
30-50 Mm
20 hours

Mesogranulation?:
7-10 Mm
2 hours

Granulation:
1-2 Mm
5 mins

Smaller stuff:
Intergranular lanes,
magnetic bright
points, diffusion


Solar Magnetic cycle 23
(EIT, LASCO & MDI Data (satellite SoHO))
http://sohowww.estec.esa.nl/
Cycle 23 Sunspot Number Prediction (March 2004)
200
150
100
50
0
1996
1998
2000
2002
2004
2006
2008
Source: NASA
2002/07/12 03:36
Source: Soho
2002/07/12 05:18
Aussois – 09/27/04


Solar Cycle 22 (Yohkoh data)
http://www.lmsal.com/SXT/homepage.html
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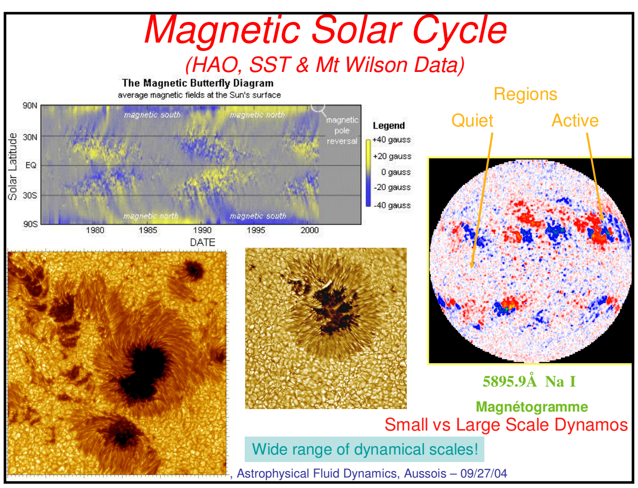
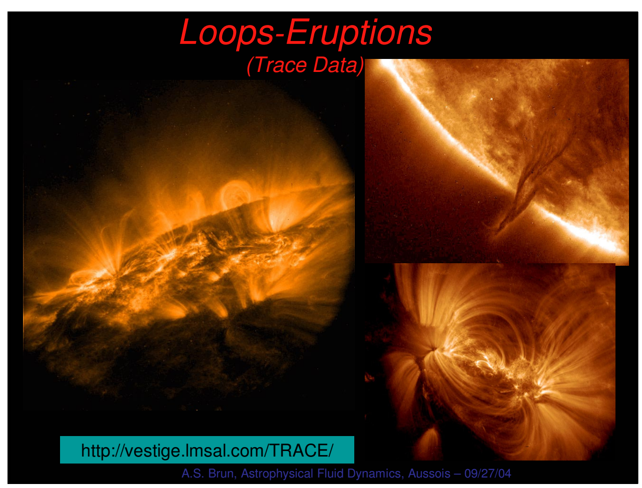

Magnetic Solar Cycle
(HAO, SST & Mt Wilson Data)
The Magnetic Butterfly Diagram
average magnetic fields at the Sun's surface
90N
30N
EQ
30S
90S
Solar Latitude
1980
1985
1990
1995
2000
DATE
Legend
+40 gauss
+20 gauss
0 gauss
-20 gauss
-40 gauss
Regions
Quiet
Active
5895.9Å Na I
Magnétogramme
Small vs Large Scale Dynamos
Wide range of dynamical scales!
, Astrophysical Fluid Dynamics, Aussois – 09/27/04


Loops-Eruptions
(Trace Data)
http://vestige.lmsal.com/TRACE/
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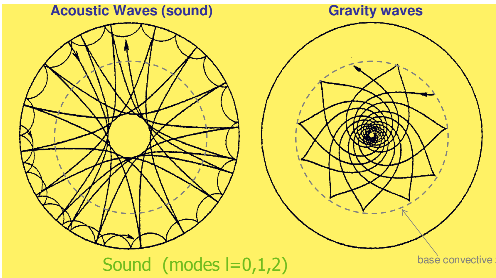
Helioseismology: The Study of Solar Waves



High frequency oscillations (~3mHz)

Low frequency oscillations (<0.4 mHz)

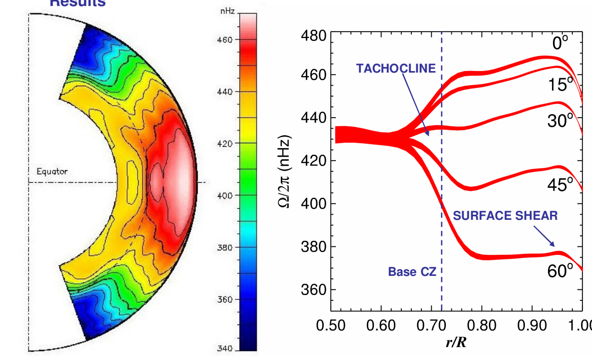
Sound Speed:
500 km/s (core)
200 km/s (base cz)
7 km/s (surface)

The Sun is BIG, it is like a SuperSuper…Bariton

Solar Internal Rotation

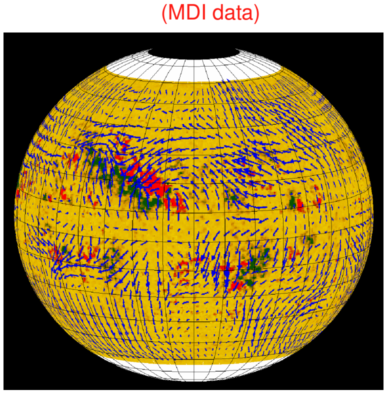
(GONG, MDI data)

Helioseismology Results



Solar SubSurface Weather

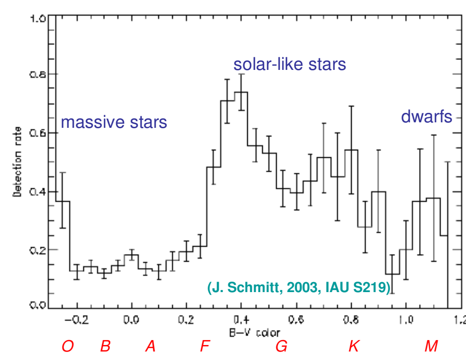
(MDI data)



(Haber et al. 2002)

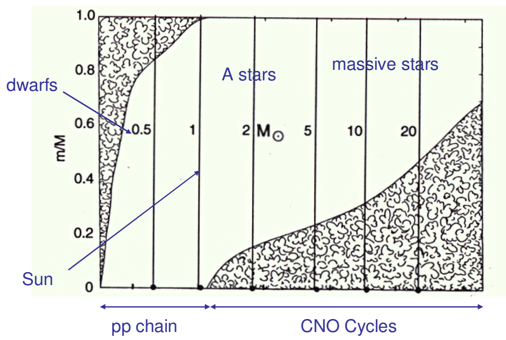
Stellar X luminosity

(ROSAT All Sky Survey)

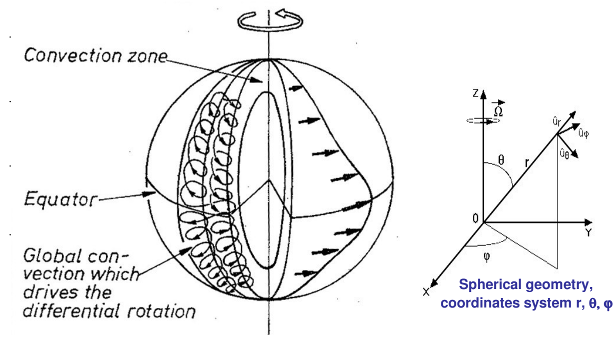


Convection in Stellar Interior

Transition between envelope and core convection: ~1.3 Msol



MHD Equations & ASH Code



3D MHD Code ASH (Anelastic Spherical Harmonics)
(Clune et al. 1999, Miesch et al. 2000, Brun et al. 2004)

3D MHD Code ASH (Anelastic Spherical Harmonics)

(Clune et al. 1999, Miesch et al. 2000, Brun et al. 2004)

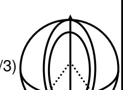
Anelastic approximation: filters sound waves but retains stratification

Poloidal-Toroidal decomposition: to conserve divergenceless ρu and B at numerical precision

LES-SGS approach: effective (turbulent) diffusivities ν, κ, η and unresolved energy Flux, $F \propto kdS/dr$

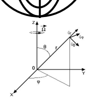
Pseudo-Spectral Method:

Horizontal Dimensions: spherical harmonics $\mathbf{Y_l^m}$ (up to $l=l_{max}=(2N_\theta-1)/3$)
Radial Dimension: Chebyshev polynomials (N_r collocation points),



Temporal Evolution:

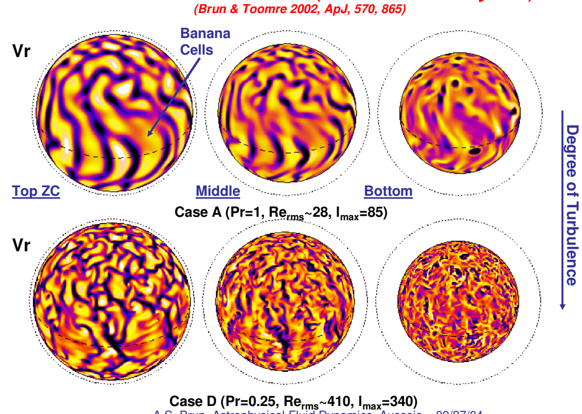
Linear Terms: 2nd order Crank - Nicholson (implicit),
NonLinear, Coriolis & Lorentz Terms : 2nd order Adams-Bashforth (explicit).

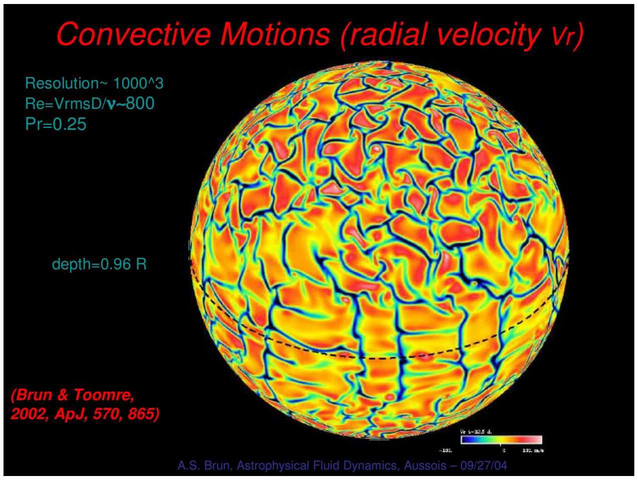


Parallelism: *Language of communication MPI*:
performance up to 120 Mflops/s per nodes on Origin2000 (Clune et al. 1999)
performance up to 250 Mflops/s per nodes on IBM SP3 (Brun & Toomre 2002).
Latest computers (HP TCS-1 & IBM SP4) are about twice faster.
Larger number of cpus used up to now: 1072

Convective Motions (radial velocity Vr)

(Brun & Toomre 2002, ApJ, 570, 865)




Convective Motions (radial velocity Vr)
Resolution~ 1000^3
Re=VrmsD/ν~800
Pr=0.25
depth=0.96 R
(Brun & Toomre, 2002, ApJ, 570, 865)
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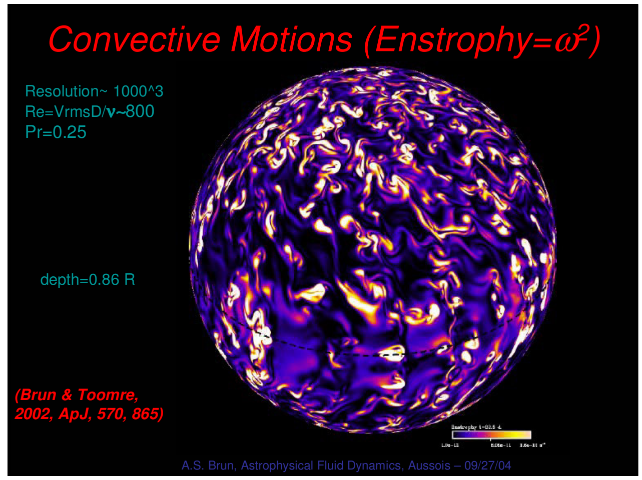
Convective Motions (Enstrophy=ω²)
Resolution~ 1000^3
Re=VrmsD/ν~800
Pr=0.25
depth=0.86 R
(Brun & Toomre, 2002, ApJ, 570, 865)
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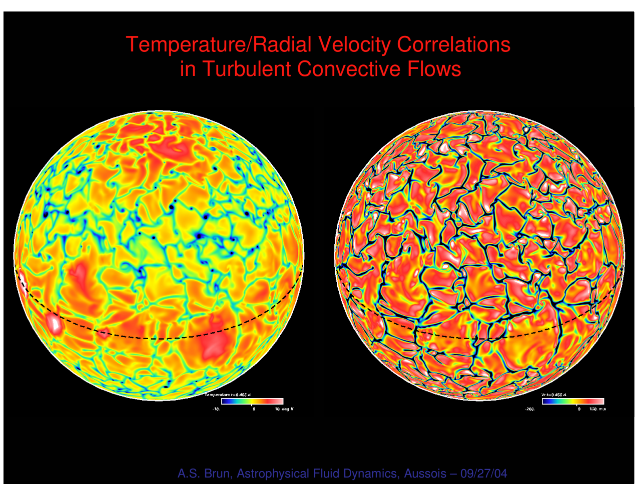
Temperature/Radial Velocity Correlations
in Turbulent Convective Flows
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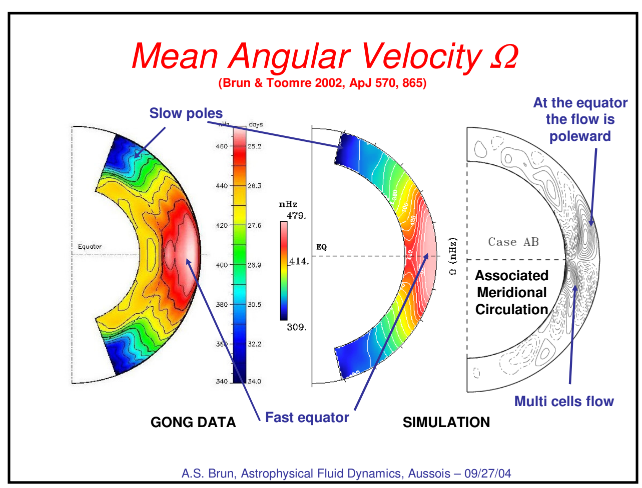
Mean Angular Velocity Ω
(Brun & Toomre 2002, ApJ 570, 865)
Slow poles
At the equator the flow is poleward
Equator
EQ
Case AB
Associated Meridional Circulation
Fast equator
Multi cells flow
GONG DATA
SIMULATION
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Angular Momentum Flux

Because of our choice of stress free boundary conditions, the total angular mometum L is conserved.
Its transport can be expressed as the sum of 3 fluxes (non magnetic case):

F_tot = F_viscous + F_Reynolds + F_meridional_circulation

Or in spherical coordinates:

F_r and F_θ are the radial and latitudinal angular momentum fluxes :

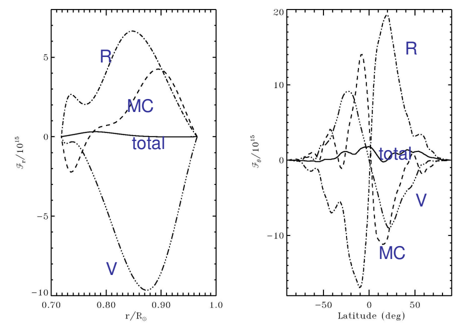
$$F_r = \hat{\rho} r \sin\theta \left[-\nu r \frac{\partial}{\partial r}\left(\frac{\hat{v}_\varphi}{r}\right) + \widehat{v_r' v_\varphi'} + \hat{v}_r \left(\hat{v}_\varphi + \Omega r \sin\theta\right) \right]$$

$$F_\theta = \hat{\rho} r \sin\theta \left[-\nu \frac{\sin\theta}{r} \frac{\partial}{\partial \theta}\left(\frac{\hat{v}_\varphi}{\sin\theta}\right) + \widehat{v_\theta' v_\varphi'} + \hat{v}_\theta \left(\hat{v}_\varphi + \Omega r \sin\theta\right) \right]$$

Transport of angular momentum by diffusion, advection and meridional circulation

Angular Momentum Balance

(Brun & Toomre 2002, ApJ, 570, 865)



The transport of angular momentum by the Reynolds stresses is directed toward the equator (opposite to meridional circulation) and is at the origin of the equatorial acceleration

Taylor-Proudman Theorem & Thermal Wind

The curl of the momentum equation gives the equation for vorticity $\vec{\omega} = \vec{\nabla} \times \vec{v}$:

$$\frac{\partial \vec{\omega}}{\partial t} + \vec{v}.\vec{\nabla}\vec{\omega} - \vec{\omega}.\vec{\nabla}\vec{v} = \nu \vec{\nabla}^2 \vec{\omega} + \frac{1}{\rho^2} \vec{\nabla}\rho \wedge \vec{\nabla} p \quad \text{(a)}$$

<u>Taylor-Proudman Theorem:</u>

In a stationary state, the φ component of (a) can be simplified to:

$$2\Omega \frac{\partial \hat{v}_\varphi}{\partial z} = 0$$ **=> vφ is cst along z**

the differential rotation is cylindrical (Taylor columns) and the flows quasi 2-D.

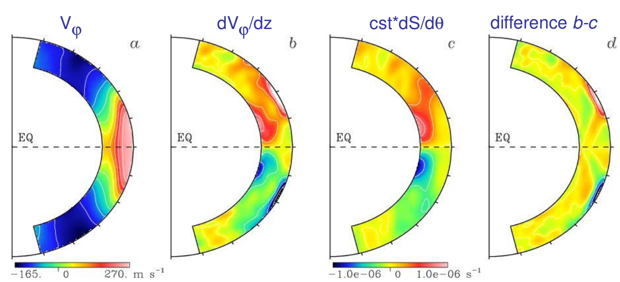
<u>Thermal Wind:</u>

The presence of cross gradient between p and ρ (baroclinic effects) can break this constraint (as well as Reynolds & viscous stresses) **:**

$$2\Omega \frac{\partial \hat{v}_\varphi}{\partial z} = -\frac{1}{\hat{\rho}^2} \vec{\nabla}\hat{\rho} \wedge \vec{\nabla}\hat{p} \Big|_\varphi = \frac{1}{\hat{\rho} C_p} \left[\vec{\nabla}\hat{S} \wedge -\hat{\rho}\vec{g}\right]_\varphi = \frac{g}{r C_p} \frac{\partial \hat{S}}{\partial \theta}$$

Baroclinicity

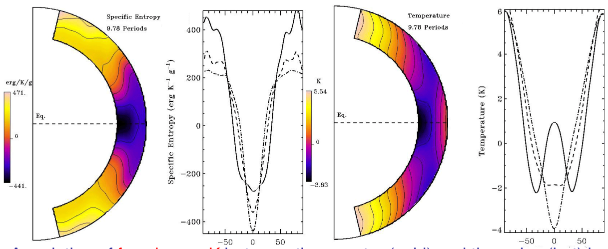
(Brun & Toomre 2002, ApJ, 570, 865)



The thermal wind contributes for some but not all of the non cylindrical differential rotation achieved in our simulation.
Reynolds stresses are the dominant players confirming the dynamical origin of Ω

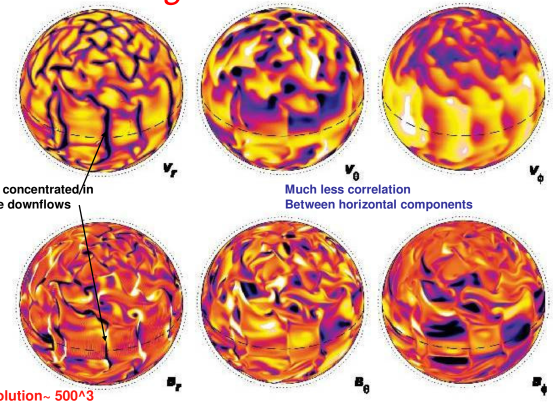
Baroclinicity

Entropy | Temperature



A variation of few degree K between the equator (cold) and the poles (hot) is established for a contrast of Ω of 30%

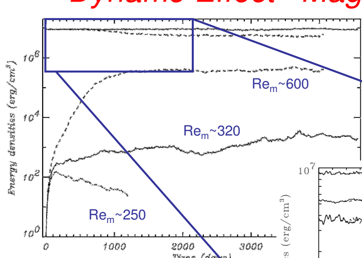
Magnetic Convection

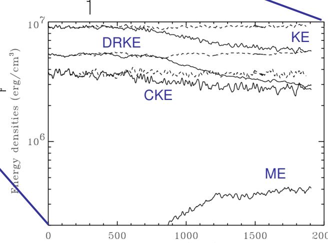


Resolution~ 500^3
Re=VrmsD/ν~150,P=0.25, Pm=4 **MAGNETIC CASE M3 (Brun, Miesch, Toomre 2004, ApJ, 614)**

Dynamo Effect –Magnetic Energy

Dynamo Threshold around Re_m=VrmsD/η~300

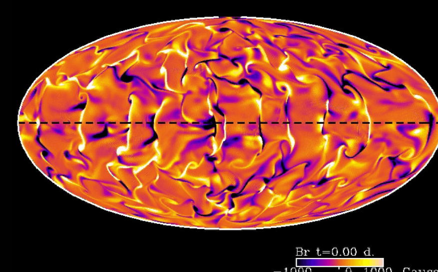




Starting from a small seed field B the magnetic energy reach a level of ~8% of KE while keeping a solar like differential rotation

ME=$B^2/8\pi$ (volume avg)
KE=$0.5\rho V^2$, DRKE=$0.5\rho\langle V\varphi\rangle_\varphi^2$
MCKE=$0.5\rho(\langle Vr\rangle_\varphi^2+\langle V\theta\rangle_\varphi^2)$,
CKE=KE-DRKE-MCKE

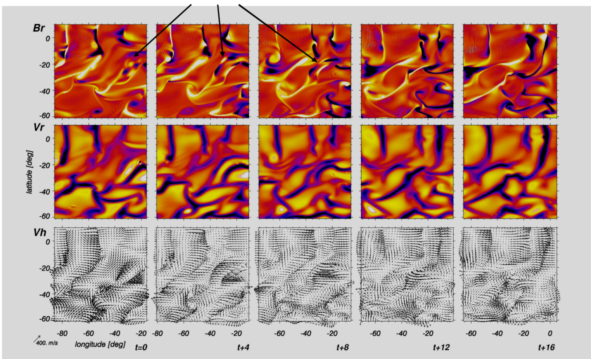
Magnetic Convection



Radial component of B

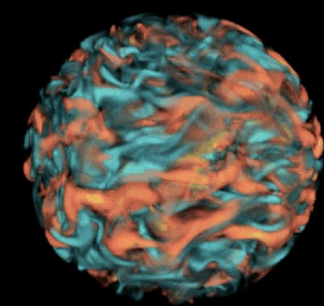
stretching and shearing of B (folding too)

Magnetic Convection



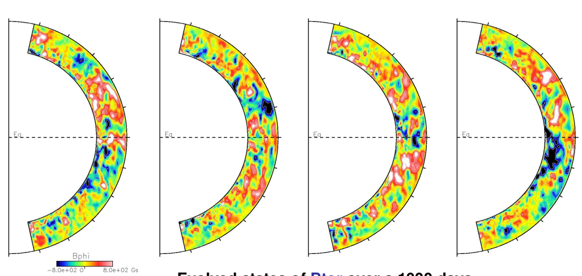
MAGNETIC CASE M3 (Brun, Miesch, Toomre, ApJ, 2004)

Magnetic Convection



3D View of Toroidal component of B (Brun et al. 2004)

Axisymmetric Toroidal Magnetic Field



Evolved states of Btor over a 1000 days

No clear systematic north-south asymmetry,
Small scale azymuthal field

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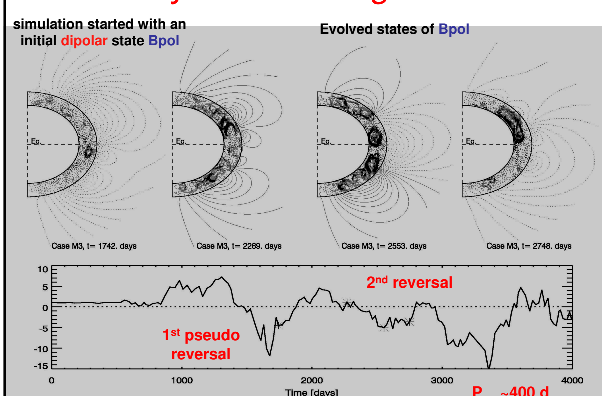
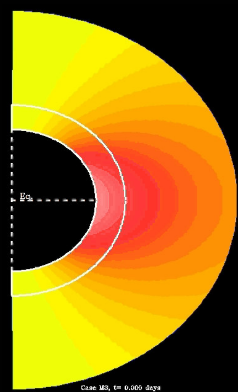
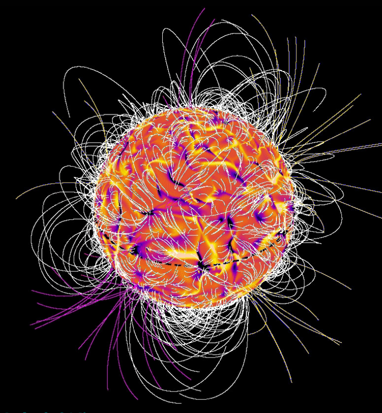
Axisymmetric magnetic fields



Axisymmetric magnetic fields



3-D Reconstruction of the Coronal Field



case M3

(Brun et al. 2004, ApJ, 614)

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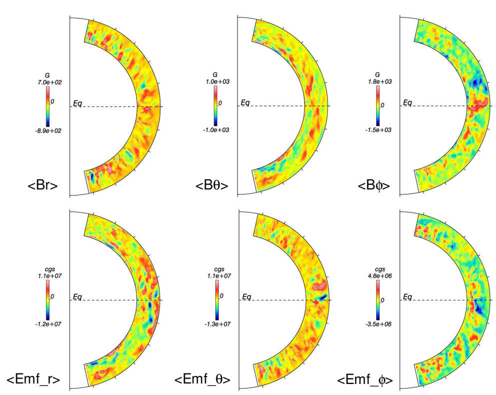
Mean Electromotive Force



No clear correspondance between $\langle u'\times b'\rangle_i$ and $\langle B_i\rangle$

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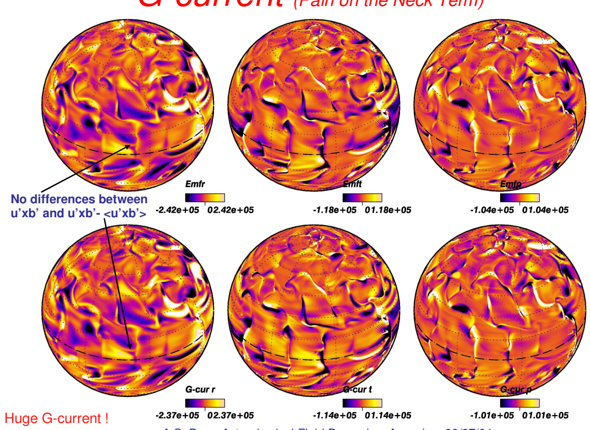
G-current *(Pain on the Neck Term)*



Huge G-current !

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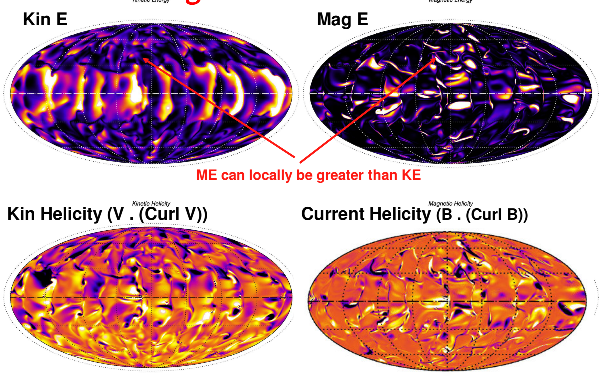
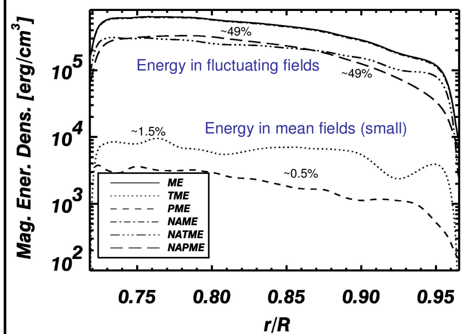
Magnetic Convection



MAGNETIC CASE M3 (Brun et al., ApJ, 2004)

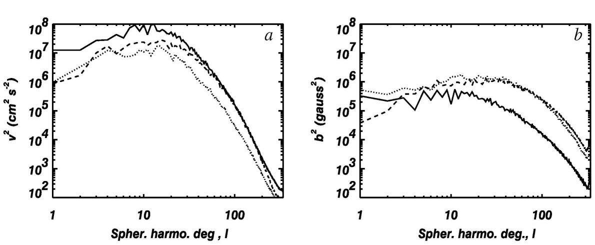
Energy Decomposition vs r



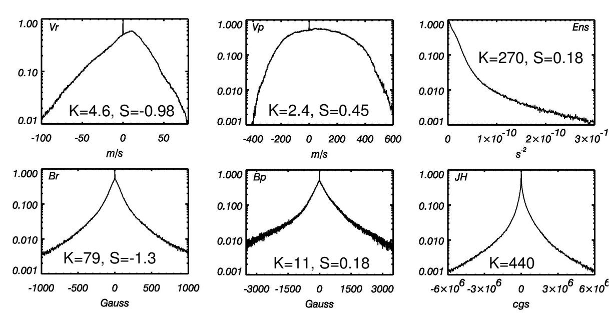
$ME=B^2/8\pi$ (horizontal avg)
$TME=\langle B\varphi\rangle_\varphi^2/8\pi$
$PME=(\langle Br\rangle_\varphi^2+\langle B\theta\rangle_\varphi^2)/8\pi$
NAME=ME-TME-PME
$NATME=(B\varphi-\langle B\varphi\rangle_\varphi)^2/8\pi$
$NAPME=[(Br-\langle Br\rangle_\varphi)^2+(B\theta-\langle B\theta\rangle_\varphi)^2]/8\pi$,

Magnetic energy peaks at the bottom of the shell, due to pumping by convective plumes

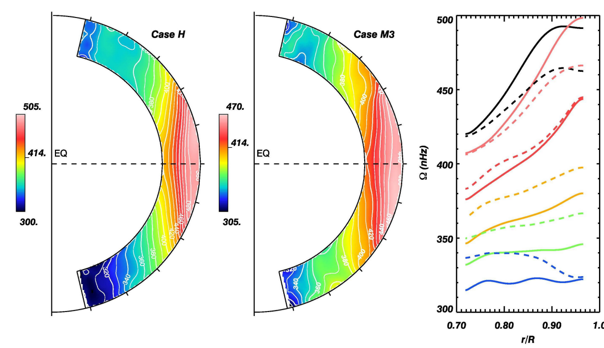
Kinetic & Magnetic Energy Spectra



Probability Distribution Functions



Mean Angular Velocity Ω



Initial state of differential rotation **Evolved state of differential rotation under the influence of the Lorentz force**

Angular Momentum Flux (MHD case)

Because of our choice of stress free and match to a Potential field boundary conditions, the total angular mometum L is conserved. Its transport can be expressed as the sum of 5 fluxes:

F_tot = F_Hydro + F_Maxwell + F_MeanB

with F_Hydro= F_viscous + F_Reynolds + F_meridional_circulation

In spherical coordinates:

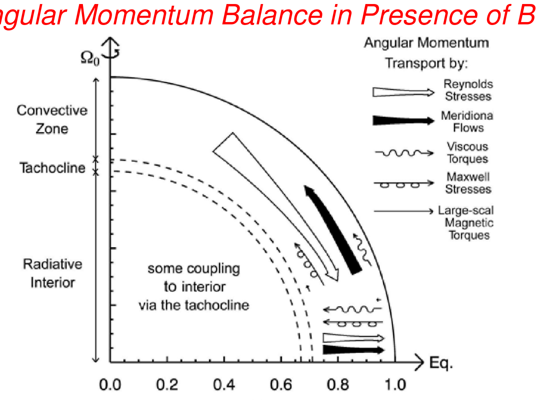
F_r and F_θ are the radial and latitudinal angular momentum fluxes:

$$F_r = \hat{\rho} r \sin\theta \left[-\nu r \frac{\partial}{\partial r}\left(\frac{\hat{v}_\varphi}{r}\right) + \widehat{v'_r v'_\varphi} + \hat{v}_r \left(\hat{v}_\varphi + \Omega_0 r \sin\theta\right) - \frac{1}{4\pi\hat{\rho}}\left(\widehat{B'_r B'_\varphi} + \hat{B}_r \hat{B}_\varphi\right) \right]$$

$$F_\theta = \hat{\rho} r \sin\theta \left[-\nu \frac{\sin\theta}{r} \frac{\partial}{\partial \theta}\left(\frac{\hat{v}_\varphi}{\sin\theta}\right) + \widehat{v'_\theta v'_\varphi} + \hat{v}_\theta \left(\hat{v}_\varphi + \Omega_0 r \sin\theta\right) - \frac{1}{4\pi\hat{\rho}}\left(\widehat{B'_\theta B'_\varphi} + \hat{B}_r \hat{B}_\varphi\right) \right]$$

Transport of ang. mom. by diffusion, advection, merid. circ., Maxwell stresses & Mean B

Angular Momentum Balance in Presence of B



The transport of angular momentum by the Reynolds stresses remains at the origin of the equatorial acceleration. The Maxwell stresses seeks to speed up the poles.

Maunder Minimum (~1650-1715)

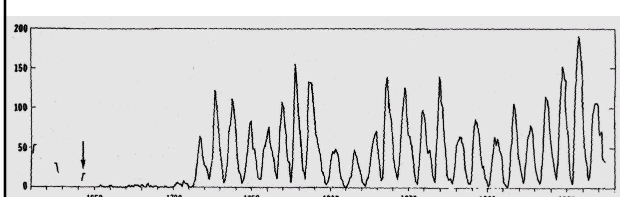
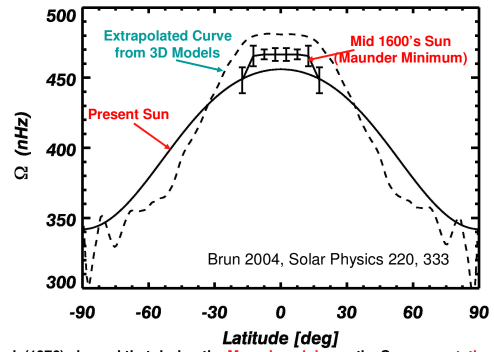


Fig. 1. Annual mean sunspot numbers, A.D. 1610–1974, from Waldmeier (1961) and Eddy (1976). Arrow marks the period of this study, 1642–1644.

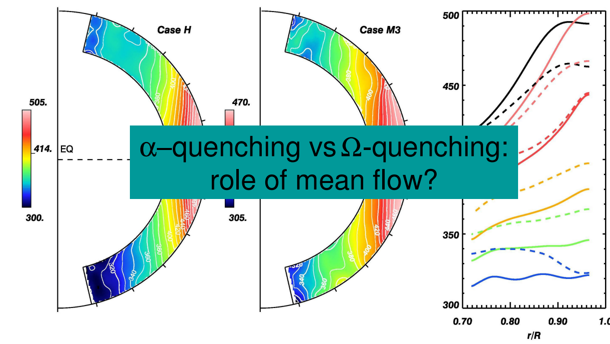
(Eddy et al 1976)

Surface Angular Velocity Ω



- **Eddy et al. (1976) showed that during the Maunder minimum the Sun was rotating 4% faster than today**
- A Magnetic energy of about 5-7% of the kinetic energy gives the correct slowing down

Mean Angular Velocity Ω



α–quenching vs Ω-quenching: role of mean flow?

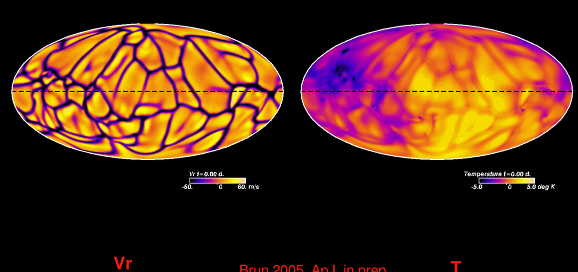
Initial state of differential rotation

Evolved state of differential rotation under the influence of the Lorentz force

Convective Motions (no rotation)

Resolution~ 400^3
Re=VrmsD/**ν**~150
Pr=0.125

depth=0.96 R

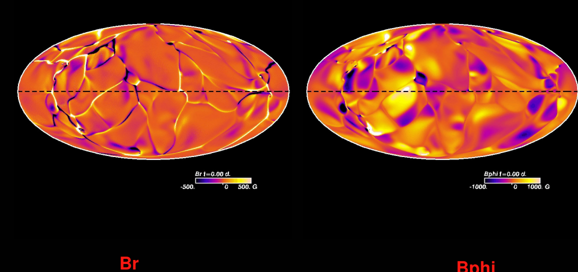


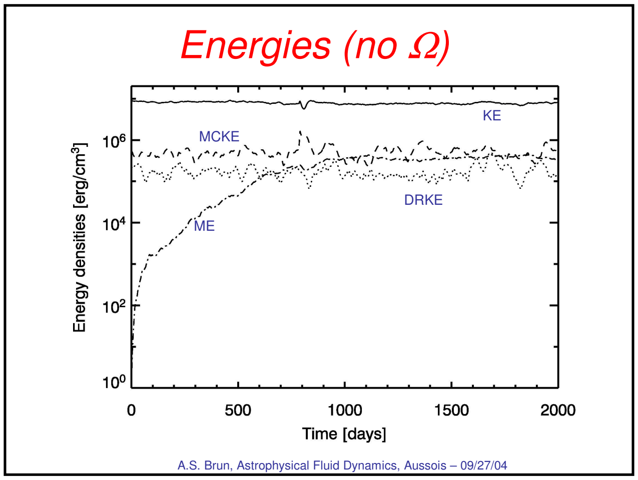
Brun 2005, ApJ, in prep

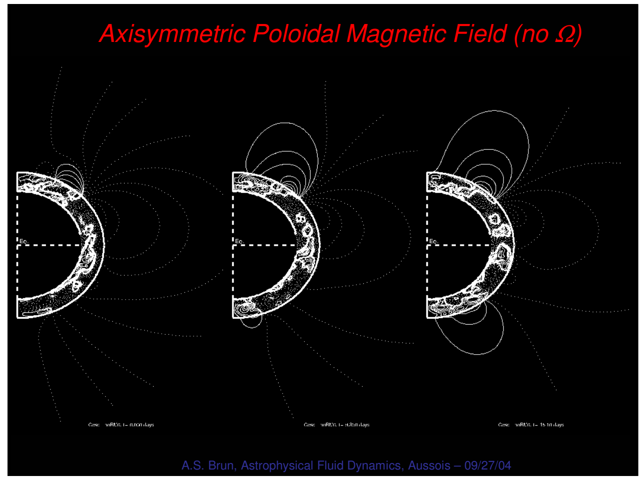
Convective Motions (no rotation)

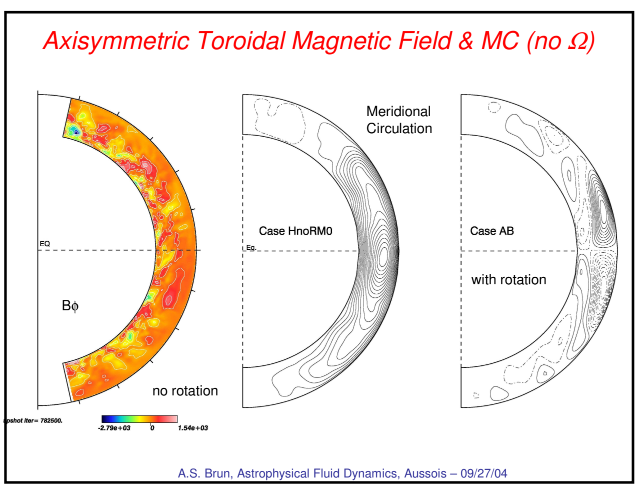
Resolution~ 400^3
Re~150
Pr=0.125, Pm=2.5

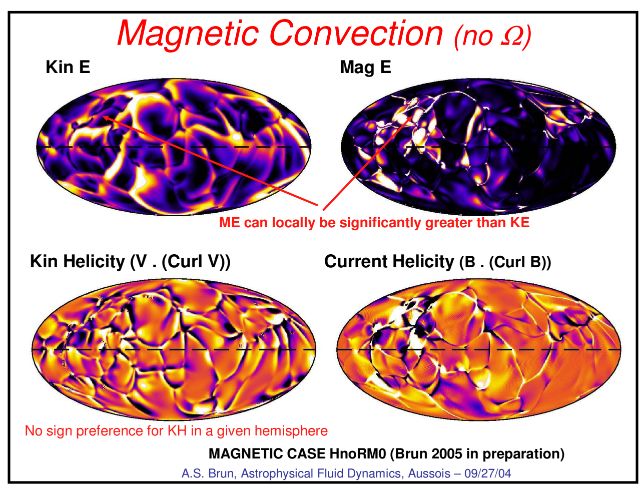
depth=0.96 R



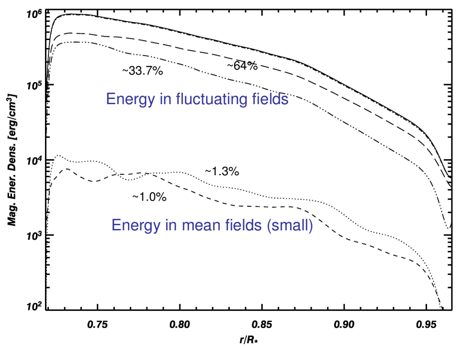

Energies (no Ω)
KE
MCKE
DRKE
ME
Energy densities [erg/cm³]
10⁶
10⁴
10²
10⁰
0
500
1000
1500
2000
Time [days]
A.S. Brun, Astrophysical Fluid Dynamics, Aussois – 09/27/04


Axisymmetric Poloidal Magnetic Field (no Ω)
A.S. Brun, Astrophysical Fluid Dynamics, Aussois – 09/27/04


Axisymmetric Toroidal Magnetic Field & MC (no Ω)
Meridional
Circulation
EQ
Bϕ
no rotation
Case HnoRM0
Case AB
with rotation
-2.79e+03
0
1.54e+03
A.S. Brun, Astrophysical Fluid Dynamics, Aussois – 09/27/04

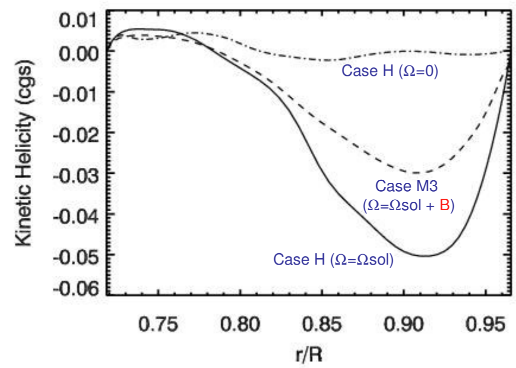

Magnetic Convection (no Ω)
Kin E
Mag E
ME can locally be significantly greater than KE
Kin Helicity (V . (Curl V))
Current Helicity (B . (Curl B))
No sign preference for KH in a given hemisphere
MAGNETIC CASE HnoRM0 (Brun 2005 in preparation)
A.S. Brun, Astrophysical Fluid Dynamics, Aussois – 09/27/04

Energy Decomposition vs r (no Ω)



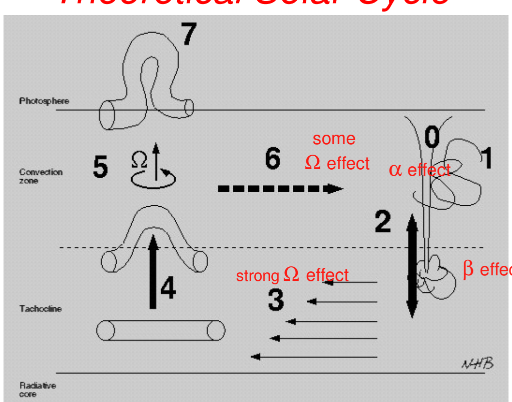
A.S. Brun, Astrophysical Fluid Dynamics, Aussois – 09/27/04

Kinetic Helicity



A.S. Brun, Astrophysical Fluid Dynamics, Aussois – 09/27/04

Theoretical Solar Cycle

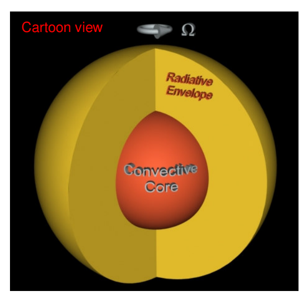


A.S. Brun, Astrophysical Fluid Dynamics, Aussois – 09/27/04

Core Convection in a $2M_{sol}$ Star

Star Properties

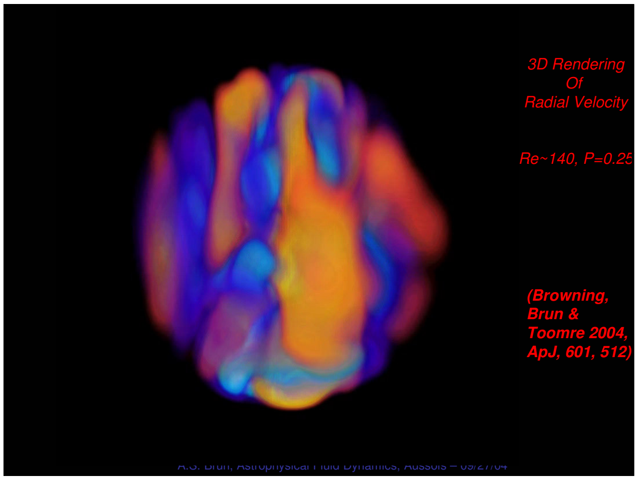
$M=2M_{sol}$, $T_{eff}=8570$ K
$R=1.9\ R_{sol}$, $L=19\ L_{sol}$
$\Omega=\Omega_{sol}$ or P=28 days

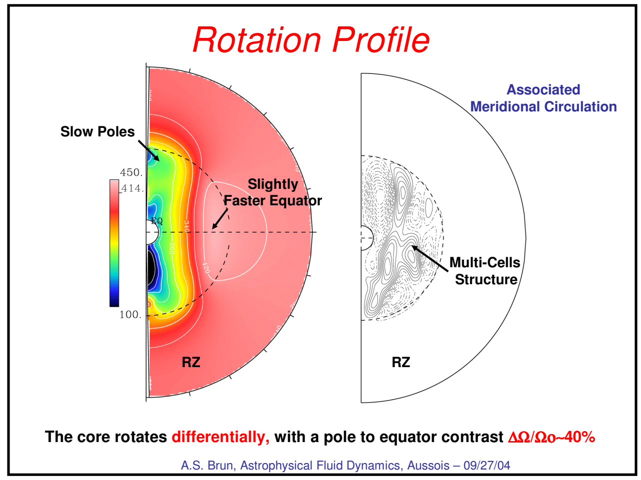


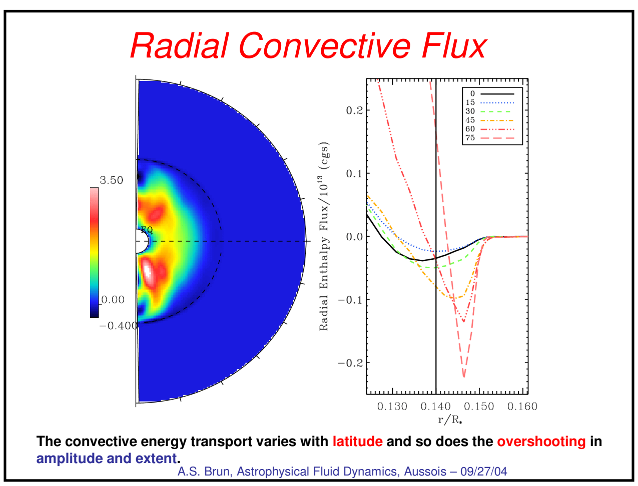
Eq of State = Ideal Gas Law
Nuclear energy source ~ $\rho\epsilon_0 T^8$
No composition gradient μ
Innermost Core r~0.02R omitted

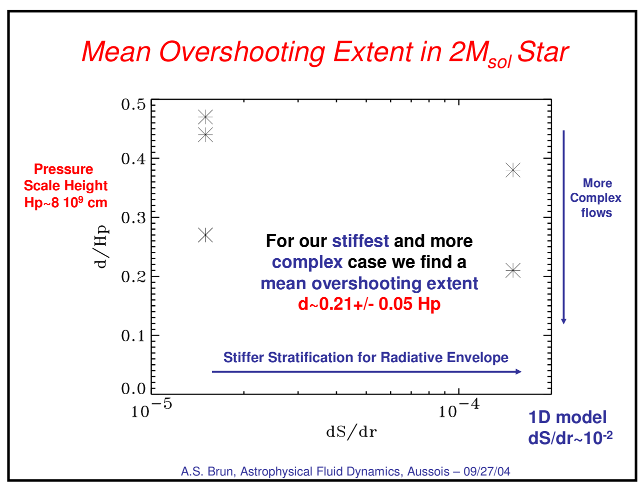
Collaboration with M. Browning & J. Toomre

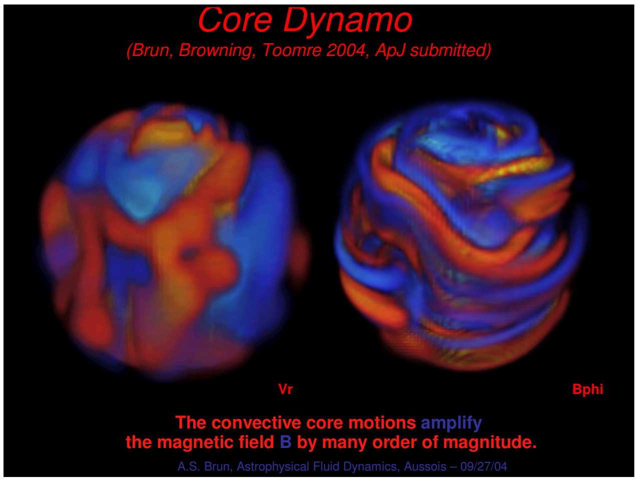
A.S. Brun, Astrophysical Fluid Dynamics, Aussois – 09/27/04

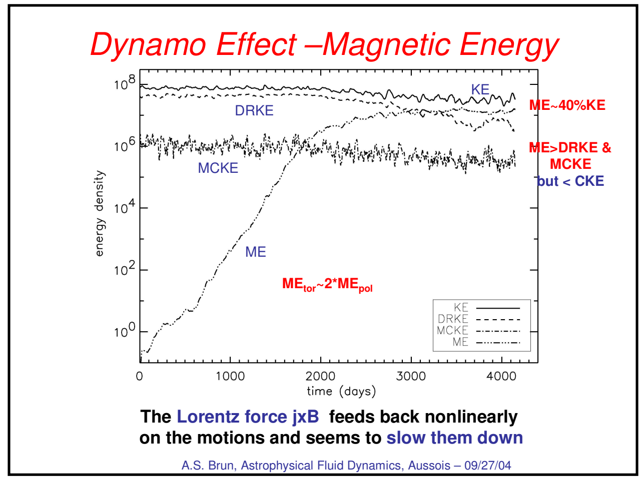

3D Rendering
Of
Radial Velocity
Re~140, P=0.25
(Browning, Brun & Toomre 2004, ApJ, 601, 512)

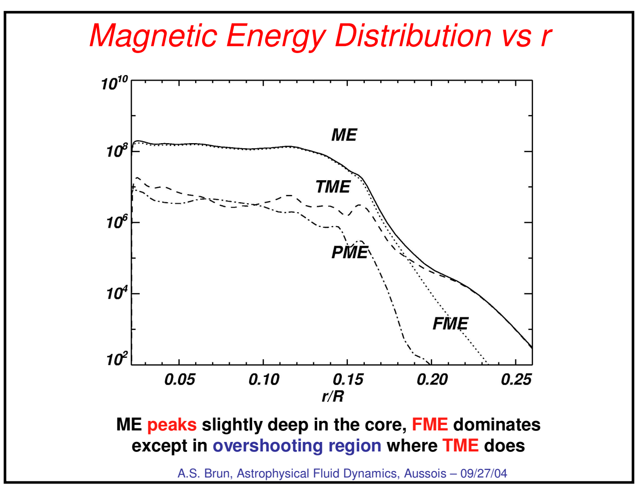

Rotation Profile
Slow Poles
450.
414.
100.
Slightly Faster Equator
Associated Meridional Circulation
Multi-Cells Structure
RZ
RZ
The core rotates differentially, with a pole to equator contrast ΔΩ/Ωo~40%
A.S. Brun, Astrophysical Fluid Dynamics, Aussois – 09/27/04

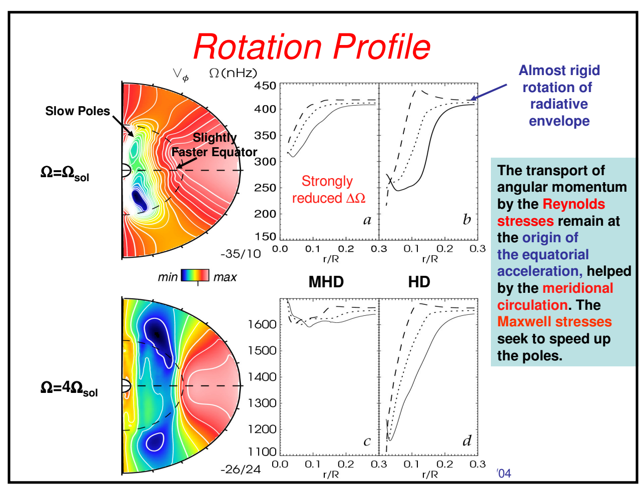

Radial Convective Flux
3.50
0.00
−0.400
Radial Enthalpy Flux/10^13 (cgs)
r/R*
0
15
30
45
60
75
The convective energy transport varies with latitude and so does the overshooting in amplitude and extent.
A.S. Brun, Astrophysical Fluid Dynamics, Aussois – 09/27/04


Mean Overshooting Extent in 2Msol Star
Pressure Scale Height Hp~8 10^9 cm
d/Hp
dS/dr
For our stiffest and more complex case we find a mean overshooting extent d~0.21+/- 0.05 Hp
Stiffer Stratification for Radiative Envelope
More Complex flows
1D model dS/dr~10^-2
A.S. Brun, Astrophysical Fluid Dynamics, Aussois – 09/27/04

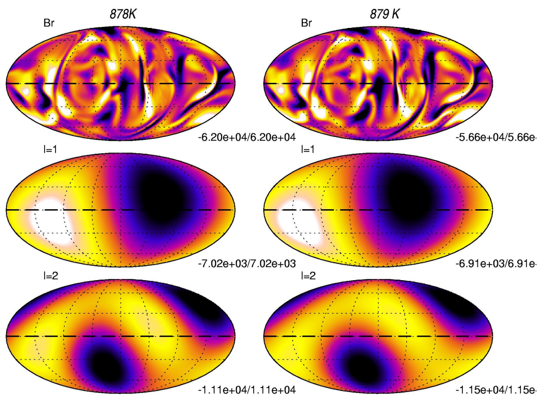

Core Dynamo
(Brun, Browning, Toomre 2004, ApJ submitted)
Vr
Bphi
The convective core motions amplify
the magnetic field B by many order of magnitude.
A.S. Brun, Astrophysical Fluid Dynamics, Aussois – 09/27/04


Dynamo Effect –Magnetic Energy
KE
DRKE
MCKE
ME
ME~40%KE
ME>DRKE &
MCKE
but < CKE
ME$_{tor}$~2*ME$_{pol}$
energy density
time (days)
The Lorentz force jxB feeds back nonlinearly
on the motions and seems to slow them down
A.S. Brun, Astrophysical Fluid Dynamics, Aussois – 09/27/04

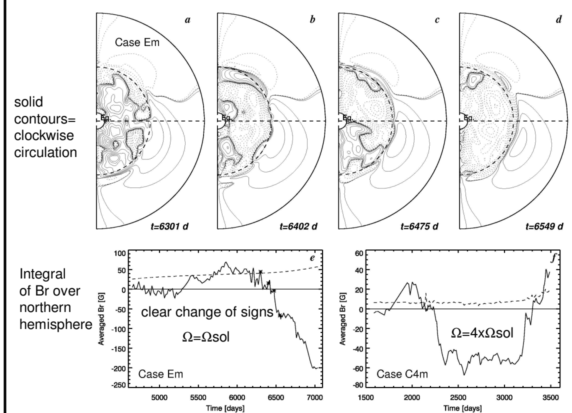

Magnetic Energy Distribution vs r
ME
TME
PME
FME
r/R
ME peaks slightly deep in the core, FME dominates
except in overshooting region where TME does
A.S. Brun, Astrophysical Fluid Dynamics, Aussois – 09/27/04


Rotation Profile
Slow Poles
Slightly
Faster Equator
Ω=Ω$_{sol}$
Ω=4Ω$_{sol}$
Strongly
reduced ΔΩ
MHD
HD
Almost rigid
rotation of
radiative
envelope
The transport of angular momentum by the Reynolds stresses remain at the origin of the equatorial acceleration, helped by the meridional circulation. The Maxwell stresses seek to speed up the poles.
'04

Core Dynamo: Drifting Dipole

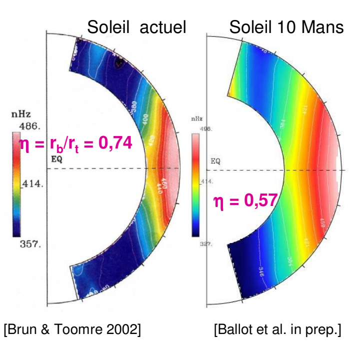


Core Dynamo: Polarity Reversal



3-D HD Models of Young Suns

(collaboration J. Ballot & S. Turk-Chièze)

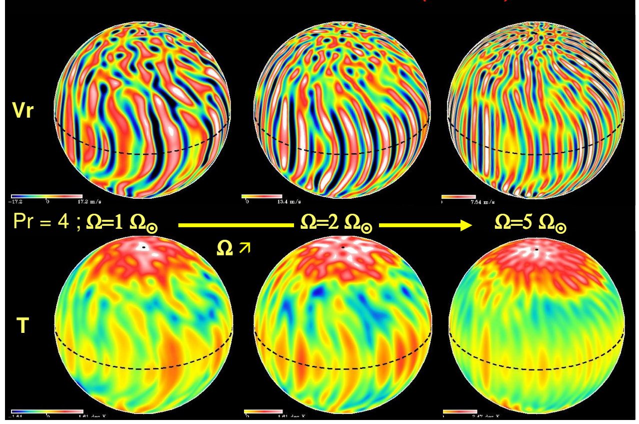


[Brun & Toomre 2002] [Ballot et al. in prep.]

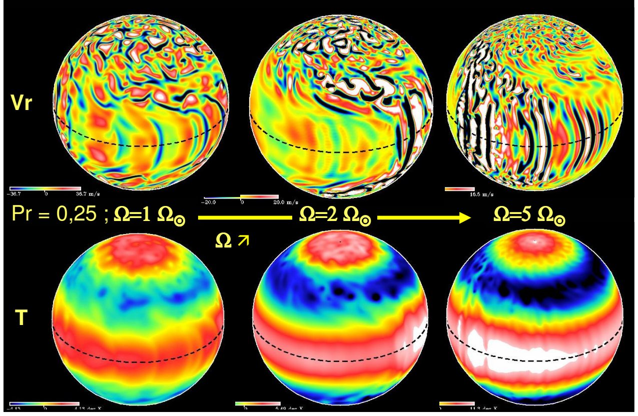
- **Convection in a spherical shell, 1M$_\odot$ star, 10 Myr**
- **Rotation rate: Ω = 1, 2, 5 $\Omega_\odot$**
- **1-D structure from CESAM**
- **Thicker shell and larger density contrast (~60)**

- **Parameter space(Ω, κ_{eff}, ν_{eff})**
- **Explored with Re ~ cste & Pr = ν/κ = cste**

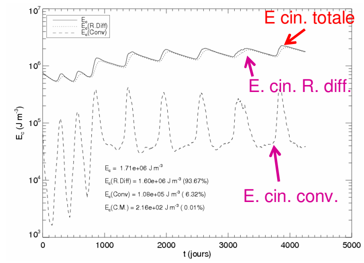
Effects of Rotation (Pr>1)



Effects of Rotation (Pr<1)



Spatial and Temporal Intermittency

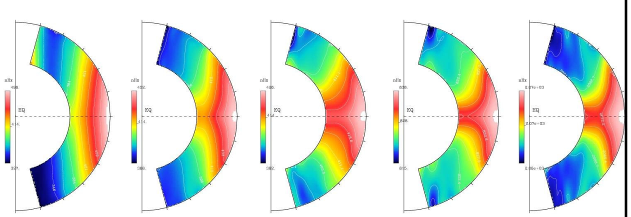


Pr ↘ et Ω↗
Forte intermittence spatiale
➔ convection turbulente « localisée »

➔ Régime « oscillant » : « localisé » dans l'espace des paramètres. Probablement dépassable vers un état chaotique.
[Gröte & Busse 2001]
Rmq : cependant ΔΩ/Ω peu variable…

Cas Pr=0.25 & **Ω=5 Ω☉**

Differential Rotation vs Ω

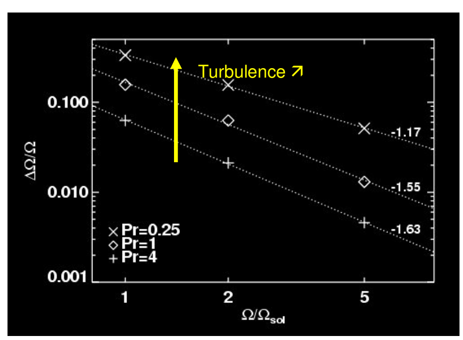


Pr = 0,25 ← **Pr = 4 ; Ω=1 Ω☉** → **Ω=5 Ω☉**

ΔΩ↗ **ΔΩ↘**

Ballot, Brun & Turck-Chieze 2004, in prep

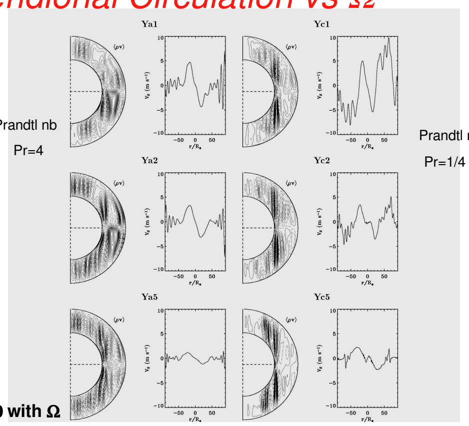
Differential Rotation vs Ω



Decrease of ΔΩ/Ω and ΔΩ with Ω
(However with Pr=0.25, ΔΩ ~ cste)

Ballot, Brun & Turck-Chieze 2004, in prep

Meridional Circulation vs Ω



Prandtl nb

Pr=4

Prandtl nb

Pr=1/4

Decrease of Vθ with Ω

Summary

- Equatorial acceleration is achieved by the transport of angular momentum via Reynolds stresses (in the regime where convective motions are dominated by rotation)
- Thermal wind balance is not the only source for the non-cylindrical rotation achieved in our simulations
- The slow pole behavior of the solar angular velocity Ω seems to be correlated with a weak meridional circulation at high latitudes
- The meridional circulation is found to have a multi-cells structure at odd with the profile used in most mean field dynamo models (mostly Babcock-Leigthon type)

Summary

- A non linear dynamo regime can be sustained for magnetic Reynolds number Rm~ 300
- It appears that Maxwell stresses seek to speed up the pole (large scale magnetic torques are very small)
- Fields reversals can occur but there are too fast due to the lack of a tachocline (stable layer)
- A ratio ME/KE of 5 to 7% in the Sun leads to the correct damping of the differential rotation seen between Maunder minimum and today's Sun
- A dynamo can occur even without rotation, changes the ratios between DRKE,MCKE, TME & PME

Summary

Core Convection and Dynamo:

- Strong retrograde differential rotation
- Large amplification of B, up to equipartition
- Strong feed back of Lorentz forces (ME/KE>40%)

Young Sun and fast rotation:

- Fast rotation leads to spatial and temporal intermittency in convection
- $\Delta\Omega/\Omega$ vs Ω decreases slightly faster than $1/\Omega$
- Meridional circulation amplitude is found to decrease with Ω