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Nonlinear simulations of magnetic instabilities in stellar radiation zones: The role of rotation and shear

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Using the 3-dimensional ASH code, we have studied numerically the instabilities that occur in stellar radiation zones in presence of large-scale magnetic fields, rotation and large-scale shear. We confirm that some configurations are linearly unstable, as predicted by Tayler and collaborators, and we determine the saturation level of the instability. We find that rotation modifies the peak of the most unstable wave number of the poloidal instability but not its growth rate as much as in the case of the $m = 1$ toroidal instability for which it is changed to $\sigma = \sigma_{\rm A}^2/\Omega$. Further in the case with rotation and shear, we found no sign of the dynamo mechanism suggested recently by Spruit even though we possess the essential ingredients (Tayler's $m = 1$ instability and a large scale shear) supposedly at work.

1 Introduction

The solar magnetic field is dominated by magnetic field generated through dynamo action in the upper layer of our star (Ossendrijver 2003; Brun, Miesch & Toomre 2004). However little is known about the field topology and strength deep within the Sun, in its radiative interior. Large scale magnetic fields are known to be linearly unstable. These instabilities have been studied in detail by Tayler and his collaborators in the 70's and 80's (Tayler 1973; Markey & Tayler 1973, 1974; Tayler 1980; Pitts & Tayler 1985) and by Wright (1973). More recently, these instabilities have been summarized by Spruit (1999, 2002), who also suggested that a fossil magnetic field permeating the radiation zone of a rotating star possessing a large scale shear could even trigger a dynamo mechanism, and maintain a level of turbulence that causes the mixing of chemical elements. A crucial point in the description of this mechanism is to determine the saturation level of the field in the non-linear regime, and to verify whether the field is regenerated; this lies beyond analytical treatment and can only be achieved through nonlinear numerical simulations.

We have carried out such simulations for the primary purpose of checking whether a fossil magnetic field can prevent the radiative spread of the solar tachocline (Brun & Zahn 2006), as was claimed by Gough & McIntyre (1998). We found that it was very difficult to prevent the poloidal field lines from connecting with the top latitudinal shear and as a direct consequence from imposing Ferraro's law of isorotation in the whole radiative interior. One striking result that was possible to get thanks to our three-dimensional code ASH, is the existence in our simulation of non axisymmetric MHD instabilities. Effectively we were able to capture Tayler's instabilities of both the poloidal and the toroidal magnetic field configuration. We thus decided to study them in the non-linear phase of their growth (Brun & Zahn 2006) and in a very recent work (Zahn, Brun & Mathis 2007) we have discussed the dynamo loop proposed by Spruit and modeled in a simple cylindrical configuration by Braithwaite (2006). We have found that it is not as straightforward as these two authors suggest to actually close the dynamo loop with just a large-scale shear and Tayler's toroidal instability (cf. Fig. 2 of Zahn et al. 2007). Clearly nonlinearities have to play a major role in order to regenerate the mean ($m = 0$) poloidal field that the large scale shear has to shear such as to sustain Tayler's toroidal $m = 1$ instability and thus dynamo action. However in our simulations with the ASH code we have not yet been able to regenerate the mean poloidal field, even though we have reached relatively high magnetic Reynolds number ($\sim 10^5$), and as a consequence we do not find dynamo action (see Gellert et al. 2007 for a similar result using a geometry closer to that of Braithwaite). More work remains to be done in order to clarify the conditions for which dynamo action in stellar radiation zones can occur.

In Brun & Zahn (2006) and Zahn et al. (2007), we have seen the development of the Tayler instabilities in presence of rotation and of a latitudinal shear imposed at the top of the tachocline. Here we report the results of a more recent numerical study, in which we have computed non rotating and rotating models of the solar radiative zone, but without imposing the latitudinal shear as in the tachocline runs. In Sect. 2 we briefly present the numerical model, in Sect. 3 we discuss the differences between non rotating, rotating and tachocline runs and conclude in Sect. 4.

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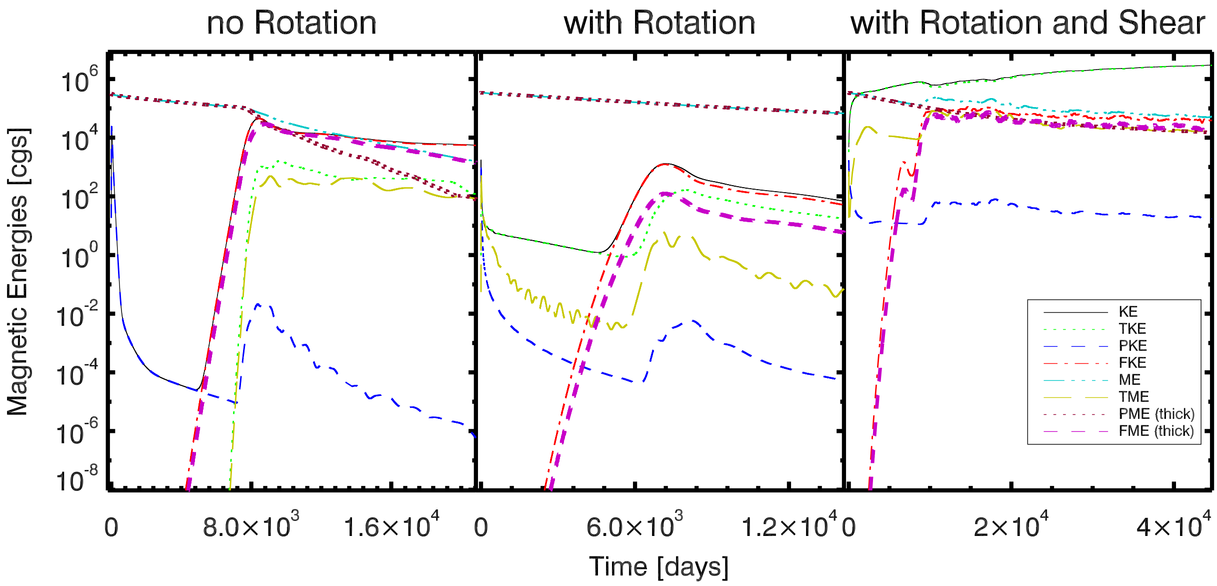

Fig. 1 (online colour at: www.an-journal.org) *Left*: Temporal evolution of the ME & KE in the non rotating case. Note the fast growth of FME & FKE and the rapid (but delayed) rise of TME & TKE. *Middle*: the case with rotation. Again FME & FKE grow fast but saturate at a lower level. *Right*: the tachocline case. We clearly see the 2 phases of non-axisymmetric instability (characterized again by the fast increase of FKE and FME): the first, for $t < 7000$ days, is associated with the unstable configuration of the dipolar field as with the two other cases, and the second, for $t > 7000$ days, with the toroidal field produced by winding up the poloidal field through the differential rotation.

2 The Numerical model

We make use of the numerical code ASH (Anelastic Spherical Harmonic; see Clune et al. 1999) which was originally designed to model the solar convection zone (Brun & Toomre 2002). It has since been extended to include the magnetic induction equation and the feedback of the field on the flow via Lorentz forces and Ohmic heating (see Brun, Miesch & Toomre 2004 for more details) and adapted to model stellar radiation zones (Brun & Zahn 2006). This code solves the full set of 3-D MHD anelastic equations of motion (Glatzmaier 1984) in a rotating spherical shell on massively-parallel computer architectures. Here we apply it to examine the nonlinear evolution of Tayler's non axisymmetric instabilities in the presence or absence of rotation and of a latitudinal shear. The computational domain extends from $r_{\rm bot} = 0.35\,{\rm R}_\odot$ to $r_{\rm top} = R = 0.72\,{\rm R}_\odot$; we thus focus on the bulk of the stably stratified zone, excluding the nuclear central region, and we ignore its possible back reaction on the convective envelope. We refer the reader to Brun & Zahn (2006) for further information on the equations that are solved, their boundary conditions and the numerical method employed. To enable us to resolve the smallest scales, the diffusion coefficients (viscosity, thermal and magnetic diffusivities) were all increased with respect to their solar values, but we took care to respect their hierarchy. The time-step was chosen such as to accommodate the Alfvén crossing time. In the cases discussed here, we imposed initially a purely poloidal field of dipolar type, which was buried below $r_{\rm bot} = 0.64\,{\rm R}_\odot$; its strength of about 3 kG was taken so that the magnetic torque could balance the advection of angular momentum through the tachocline circulation (cf. Brun & Zahn 2006). Three cases have been performed that all start with the same field strength and topology but differ by having or not a rotating reference frame or the presence of an imposed large scale shear.

3 MHD instabilities in radiation zone and the role of rotation and shear

The instabilities are best described in following the temporal evolution of various components of the kinetic (KE) and magnetic (ME) energies, averaged over the whole computational domain (see Fig. 1). PKE & PME designate the mean (axisymmetric) poloidal components of respectively KE and ME, TKE & TME their mean toroidal components, and FKE & FME their non-axisymmetric components (see Brun et al. 2004 for their analytic expressions). In the left panel of Fig. 1, we present the temporal evolution of KE and ME and their components for the non rotating case. We observe the fast, exponential growth of both FKE and FME by many orders of magnitude. The e-folding time is ≈ 120 days and the saturation of the fluctuating energies occurs around 8000 days, where it reaches almost the level of PME, meaning that most of the energy of the poloidal field has been drained to fuel the instability. In the process, some of the fluctuating field has been transformed into mean toroidal field TME, through non-linear self-interaction as revealed by its growth-rate, which is about twice that of FME. Likewise, FKE saturates since the velocity fluctuations are in-

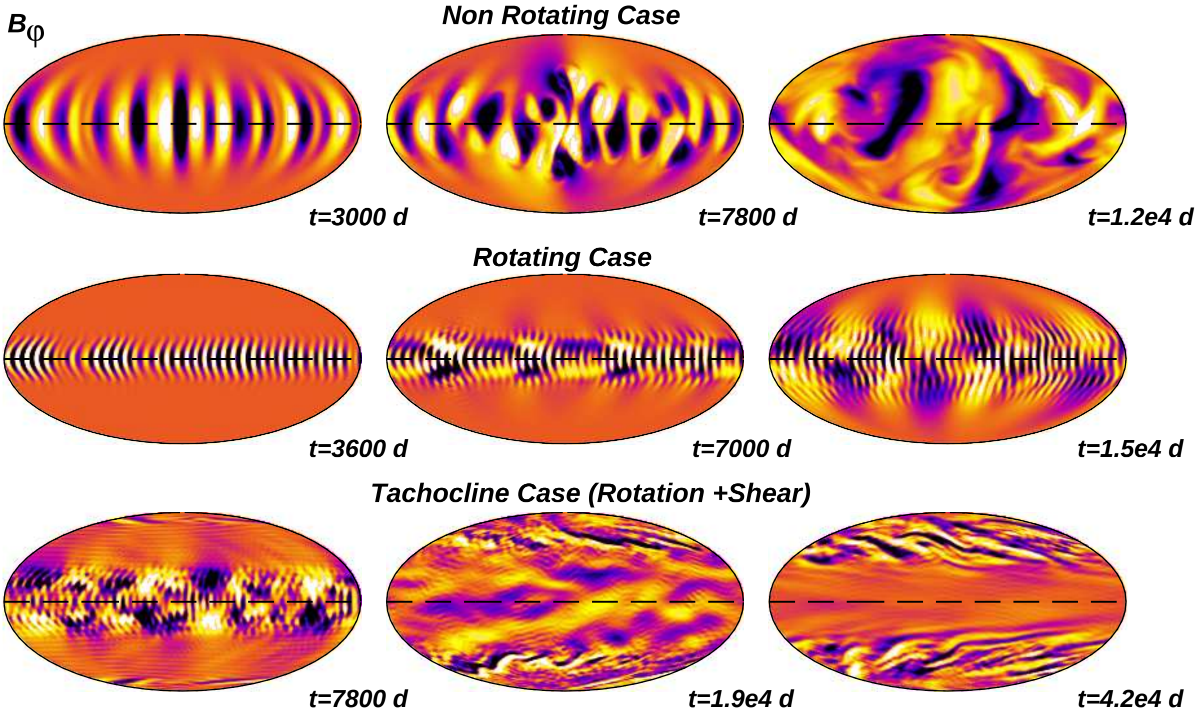

Fig. 2 (online colour at: www.an-journal.org) Temporal evolution of the non axisymmetric longitudinal field (azimuthal average, $m = 0$, has been subtracted off) for the non rotating case, the rotating case and for the tachocline case with rotation and shear. Note how the nonlinear evolution of the instability differs in the presence of large scale shear, with B_ϕ taking a ribbon like shape.

timately related to the magnetic field fluctuations and TKE grows at a rate similar to TME. PKE starts growing when FKE has significantly rose and reached a level of energy of similar amplitude, but it then saturates much faster and declines. Note however that the decay of FKE and FME differs, which is certainly due to the fact that our run uses a magnetic Prandtl number of Pm $= \nu/\eta = 10^{-1}$, i.e larger magnetic diffusion than viscous dissipation. The triggering of the poloidal instability seems to be linked to the fact that the field is seeking a more stable configuration, involving both poloidal and toroidal components. Markey & Tayler (1973) indeed showed that a toroidal field that reach an amplitude of the same order than that of the poloidal field will make the magnetic field configuration stable. After saturation, since most of the magnetic energy is now in form of the fluctuating field, its Ohmic decay proceeds on a faster pace than that of PME in the early phase. Characterizing now the spatial properties of the instability, we see that the fluctuations are first located at low latitude and at the bottom of the domain (see Fig. 2 top left). We conclude that it is one of the instabilities described in the pioneering works quoted above, which occurs when the field is purely poloidal. When analyzing the energy spectrum in m of the fluctuations of the azimuthal field B_ϕ, one sees clearly that the maximum growth occurs at rather high azimuthal wavenumber: $m \approx 12$. This is in agreement with Wright (1973) and Markey & Tayler (1974), who found that the growth-rate of the instability should increase with m in the non dissipative case. Our simulations include dissipative effects and thus we expect to find the maximum growth rate at an intermediate m rather than at $m = m_{\max} = 85$. Quickly after, the instability starts filling up the whole sphere (Fig. 2 top middle and top right), and progressively the maximum amplitude of the spectra shifts toward lower m and ℓ as the dissipation of the smallest scales starts kicking in. Note in the middle panel of the top row of Fig. 2, how the instability develops, by having patches of positive and negative polarity starting to rotate and entrain one another, not unlike interchange instabilities. There is no sign of a dynamo in this simulation, which is not surprising since there is no energy available here to feed the process: all components of the magnetic field and of the velocity fields end up decaying away.

Considering now the rotating case, we notice in Fig. 1 (middle panel) that again FME and FKE rise by many orders of magnitude with an e-folding of about 90 days. We can conclude that for the poloidal instability, rotation does not seem to delay the triggering of the instability and actually make the instability rise faster. This is surprising because rotation usually decreases the growth rate of the instability when compared to a non rotating case. Pitts & Tayler (1985) actually showed that this is particularly true for the $m = 1$ instability of a toroidal field. They also showed that for a purely poloidal configuration, the stabilization and reduction of the growth rate by rotation is not as important as in the purely toroidal configuration, in particular for high

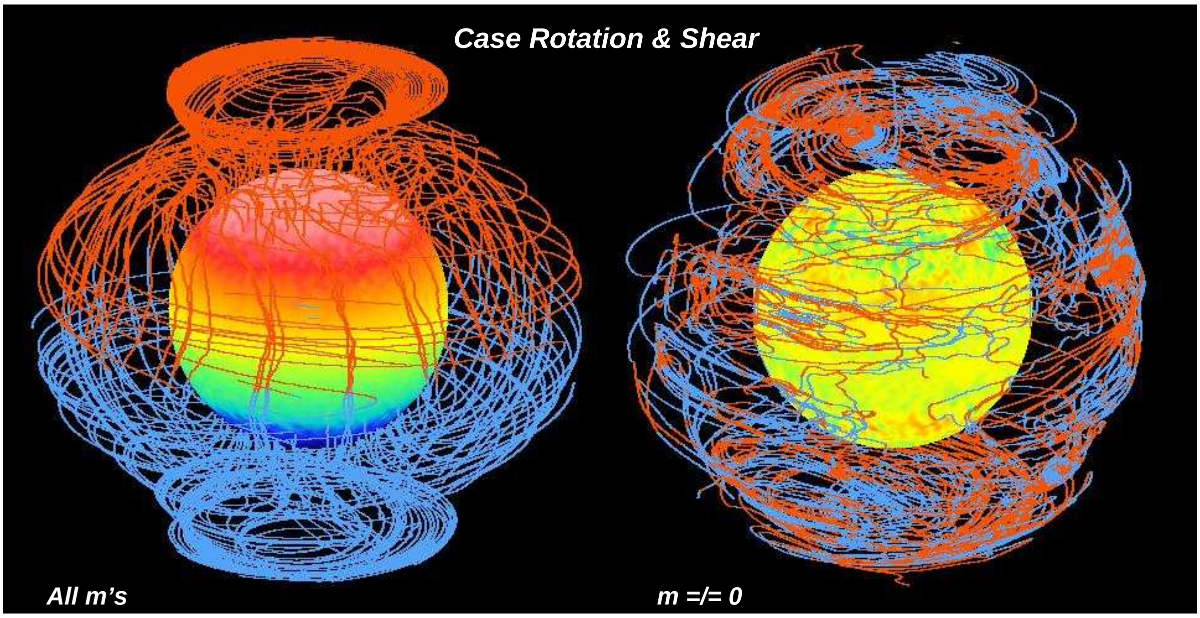

Fig. 3 (online colour at: www.an-journal.org) 3-D rendering of the magnetic field lines in the radiative interior of the case including rotation and shear at the peak of the instability $t \sim 10^4$ days. On the left we show all the m's whereas on the right we have subtracted the $m = 0$ component of the three components of the field. The fields are red (blue) when pointing towards (away from) the viewer. The background image is located at $r = 2.5 \times 10^{10}$ cm and represent the radial component of the field.

wave number disturbances. A spectral analysis of the instability in the rotating case confirms their analysis, since in this case the instability actually possesses a much higher dominant m, close to 40, compared to the non rotating case where it is around 12. Since high m disturbances of purely poloidal field are less sensitive to rotation, it can explain why we do not see much difference in the growth rate of the poloidal instability. However we do see differences between the non rotating and the rotating case when comparing the left and middle panel of Fig. 1. We note that FME in the rotating case saturates at a much lower level (about 1000 times smaller) than in the non rotating case. Further, PME is not as affected as in the non rotating case (no slope change in the curve), even though it still plays the role of energy reservoir for the instability. Certainly rotation modifies the development of the instability since all the energies reach a lower level of saturation. This is certainly linked to the fact that there is now a preferred direction along the rotation axis and the instability can not fully develop as freely in the whole sphere, thus occupying less volume. This is clearly seen in Fig. 2 (middle row), where we again show the temporal evolution of the longitudinal component of the magnetic field after having subtracted the axisymmetric ($m = 0$) mean. The instability starts at the bottom of the shell near the equator and develops mostly near the equatorial region. After saturation, which occurs around 7000 days we also find as in the previous case that the instability decays away when the magnetic field has reached a more stable configuration.

We finally turn to the last case possessing both rotation and shear (Fig. 1, right panel). From the onset, as in the two previous cases, FKE & FME rise exponentially by many orders of magnitude at a fast pace (the e-folding time is ≈ 90 days). They saturate around 7000 days at a level similar to the purely rotating case, about 1000 times below the energy of the mean poloidal field (PME). Interestingly enough we find that both rotating cases possess the same growth rate and saturation level for FKE and FME which is set by the high m poloidal instability as discussed previously. Again the fluctuations are located at low latitude and at the bottom of the domain (Fig. 2 bottom left) and peaked as in the rotating case around $m = 40$. Thus the introduction of a large scale shear does not seem to modify the poloidal instability, which after saturation starts decaying as in the purely rotating case. However it clearly changes the very late evolution of the simulation when comparing to the purely rotating case as can be seen in Figs. 1 and 2. Indeed we can note the appearance of two bands of toroidal field of opposite polarity in the upper part of the domain (see Fig. 2 bottom/middle), one at mid latitude and the other close to the rotation axis, with opposite sign in each hemisphere. This field is generated by the shearing of the poloidal field, which very soon encounters the differential rotation spreading down from the convection zone where the latitudinal shear is imposed. At about 10^4 days the energy TME, averaged over the whole domain, matches that of the mean poloidal field, meaning that locally the toroidal field can be much stronger than the poloidal field, since it occupies a smaller volume. That toroidal field is unstable to low m perturbations, with the strongest component at $m = 1$, as predicted by the theory (Markey & Tayler 1973; Spruit 2002; Zahn, Brun & Mathis 2007). As a direct consequence of the development of the toroidal instability, FME and FKE start growing again reaching value comparable to that obtained in the non rotating case with the high m poloidal instability. Note that the mean toroidal field and the associated instability draw their energy from the differential rotation, as indicated by the conspicuous inflection of the TKE curve in Fig. 1 (right panel), near 10^4 days. Clearly the presence of a strong differential rotation modifies the nonlinear evolution of the magnetic field and its topology. In this case the magnetic field finally organizes itself into ribbon like structures near the poles due to the strong and continuous ω-effect present in this region. This is in sharp contrast with the two previous cases that do not trigger the $m = 1$ toroidal instability. As the simulation evolves, the toroidal instability develops over the entire sphere.

Thereafter all energies but one slowly decline, at a rate that is controlled mainly by the Ohmic dissipation of the mean poloidal field (PME): the e-folding time would be $R^2/2\pi^2\eta \approx 18000$ days for a uniform motionless sphere (Cowling 1957), which is compatible with what we see here. The decay of that field is particularly smooth, and there is no sign that it is regenerated or even affected by the instability. The mean toroidal field accompanies closely the decline of the mean poloidal field (see Fig. 1, right panel, and Fig. 2, bottom right), and so do the non-axisymmetric components FKE and FME, with the kinetic energy in these perturbations being about twice their magnetic energy. In the tachocline case the only exception to that overall decline is the mean toroidal kinetic energy TKE, which steadily increases as the differential rotation spreads deeper and to lower latitudes. The kinetic energy of the meridional flow (PKE) is the smallest of all energies; it is mostly concentrated toward the top of the domain and it remains almost constant during the whole evolution.

In Fig. 3, we further illustrate the topology of the magnetic field obtained in the tachocline case, by rendering in three dimensions the magnetic field lines over the whole domain. To ease the visualization we have color coded the field lines with the local sign of the radial component of the field, red corresponding to positive B_r (i.e outward). On the left panel we show the total field, including the $m = 0$ axisymmetric component, whereas in the right panel we subtracted it, such as the non axisymmetric component become more evident. The toroidal character of the field, is clearly apparent in the left panel, in particular near the polar region where displaced disk shape-like structures are evident. The initial dipolar configuration is also obvious, with field lines changing colors from red to blue when they cross the equator. The non axisymmetric field is much less organized, as is the radial direction of the field, with the sign of the fluctuating B_r changing very often. The field lines near the poles, where the shear and the toroidal $m = 1$ instability dominate, are clearly displaced with respect to the rotation axis, as anticipated by Spruit (2002).

4 Conclusion

Since we used a three-dimensional code, we were able to observe the non-axisymmetric instabilities associated with our field configurations. In the early phase as well as in the late evolution of the purely rotating case, they occur in a narrow equatorial belt at the base of the computational domain, and they are due to the purely poloidal field we have imposed initially. Later on, depending on whether there is or not a large scale shear present in the radiative stable zone, they are present either at all depths near the poles, where the shear of the differential rotation, which has spread from the top of the radiation zone, keeps generating a strong toroidal field or throughout the sphere with no latitudinal preference and a much less organized parity antisymmetry or favored direction. Their properties agree reasonably well with the predictions of Tayler and his collaborators, which were recently reviewed and completed by Spruit (1999, 2002).

An important result of our simulations is that these instabilities do not interfere with the mean poloidal field; they are able to distort the 'isogyres' (surfaces of constant angular velocity Ω), thus affecting somewhat the production of the toroidal field, but even there they seem to have limited impact. Thus we do not see a dynamo process occurring in our simulated radiative interior, as was suggested by Spruit: in all cases studied the mean poloidal field is not regenerated but it decays away through Ohmic diffusion. However it is possible that the magnetic Reynolds number required to trigger the dynamo mechanism is beyond that achieved in our simulations. We have performed two cases with magnetic Reynolds numbers of order 10^4 and 10^5 (Zahn et al. 2007), but in none of the cases we have found dynamo action. If such dynamo action occurs in radiative zone possesing large scale field and shear, the dynamo loop is harder to close than expected by Spruit (2002) and Braithwaite (2006) as discussed in length in Zahn et al. (2007, cf. their Fig. 2).

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