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## 1 Introduction

*"Convection is the least well understood physical phenomenon inside red giants"*. Such is the thought-provoking statement published in the August 2007 edition of the AGB newsletter. Turbulent convection is indeed poorly known and understood in the extended envelopes of giant stars. Its interaction with rotation that results in the establishment of meridional circulation and differential rotation needs to be more deeply understood. As of today, our understanding of stellar evolution is based on 1-D stellar evolution codes that compute the late evolution of stars on the giant branch by assuming mixing length theory (Böhm-Vitense 1958). The mixing length parameter $\alpha$ used for red giant models is the one coming from the solar calibration (Brun et al. 2002). In more sophisticated rotating evolutionary models, the transport of angular momentum in radiative regions is ensured by meridional circulation and shear-induced turbulence (Zahn 1992), while solid-body rotation is assumed in convective regions (e.g. strong turbulent diffusion of the angular momentum) in the absence of a reliable formalism to describe the transport of angular momentum in these regions. These assumptions, both on the mixing length parameter and on the angular momentum distribution inside convective regions are based on simple arguments developed many decades ago to model the Sun. However recent solar observations and models indicate that they are somewhat misleading, set aside the fact that the structure and the configuration of the convective envelope of red giant stars are completely different from that of the Sun. Indeed recent observations of red giant stars indicate that the assumption of solid-body rotation in extended low-density convective envelopes is most likely erroneous. The analysis of the rotation of horizontal branch stars in globular clusters led Sills & Pinsonneault (2000) to suggest that in order to reproduce the rotation velocity of HB stars considering that the progenitors of these stars on the red giant branch had almost zero surface rotation, a large amount of angular momentum should be retained in the inner part of the stars during the RGB ascension. Sweigart & Mengel (1979) had already reached this conclusion to explain the carbon underabundance observed in 98% of the low-mass RGB stars. This can be achieved if RGB convective envelopes rotate differentially. After confirming in Palacios et al. (2006) that assuming conservation of the specific angular momentum ($\Omega(r) \propto r^{-2}$) favours turbulent mixing below the convective zone, we have decided to investigate more deeply the interaction of turbulent convection and rotation in such an extended convective zone. In this paper we turn to 3-D simulations in order to investigate the interplay between rotation and convection in the turbulent convective envelope of a red giant star. In Sect. 2 we briefly discuss the numerical model, in Sect. 3 the patterns realized in the convective envelope, in Sect. 4 the angular momentum redistribution achieved in our simulation, and we conclude in Sect. 5.

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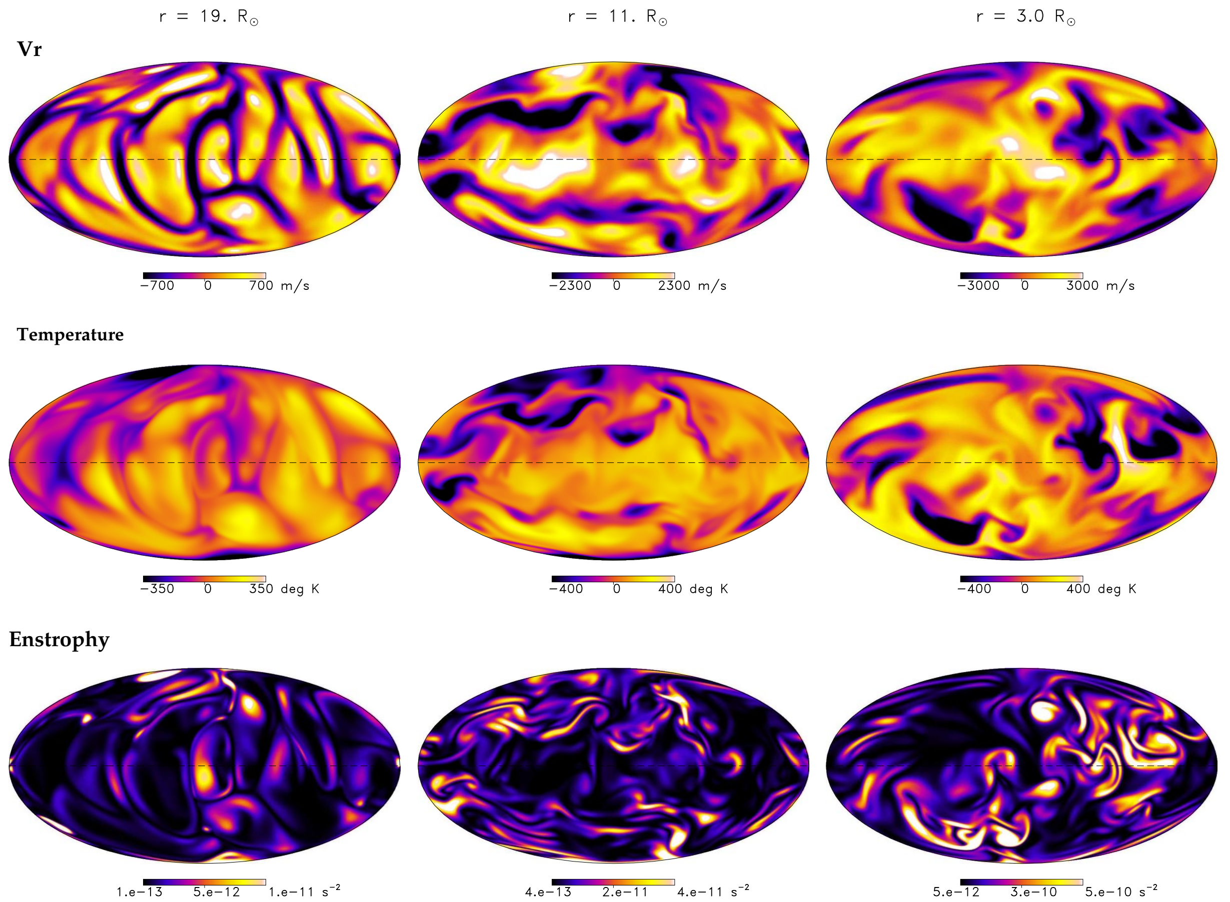

**Fig. 1** (online colour at: www.an-journal.org) Properties of the convection achieved in the shell after 4200 days presented at three depths as indicated. The radial velocity (*first row*), the temperature (*second row*) and the enstrophy (*third row*) are shown in a global view using a Mollweide projection. The geometrical factor between the different depths has been omitted.

## 2 Modelling a red giant star with ASH

We have used the Anelastic Spherical Harmonic (ASH) code in its purely hydrodynamic mode to study a low-mass RGB star. The ASH code has been applied with success to the study of both the solar convective envelope (Brun & Toomre 2002; Miesch, Brun & Toomre 2006) and of core convection in massive stars (Browning et al. 2004) and we are thus confident that it can be used to model other stars. The reader is referred to Brun, Miesch & Toomre (2004) for details on the code. Let us here briefly summarize the main characteristics of our simulation. To build the 3-D nonlinear simulation we used a 1-D red giant stellar model as initial state whose characteristics are: $M_{\rm ini} = 0.8\,{\rm M}_\odot$, $L_* = 425\,{\rm L}_\odot$ and $R_* = 40\,{\rm R}_\odot$. In order to lower the density contrast to a value that can be numerically handled by ASH (e.g. $\rho_{\rm bot}/\rho_{\rm top} = 100$), we only consider the inner $50\%$ of the convective envelope, e.g. $r \in [0.05\,R_*; 0.5\,R_*]$. This choice is also driven by the fact that getting too close to the surface would imply large Mach numbers that are not compatible with the anelastic assumption. We assume that the convective envelope initially rotates as a solid-body with a rotation rate a tenth solar ($\Omega = \Omega_\odot/10 = 2.6 \times 10^{-7}{\rm rad\,s}^{-1}$). This rotation rate corresponds to a surface equatorial velocity of about 7 km/s. We assume rigid stress free boundary conditions at the edges of the computational domain, and impose a flux of radiative energy at the base of our domain that is extracted at the surface. The simulation has been evolved over 4200 days, which represent more than 12 stellar rotations. The model is characterized by the following non-dimensional numbers, as evaluated in the bulk of the computational domain: Prandtl number Pr $= \nu/\kappa$ = 1, Reynolds number Re $= vL/\nu \simeq 400$ and Rayleigh number Ra $= 7.5 \times 10^5$.

## 3 Convective patterns

The convective instability sets in during the first hundred days of the simulation and then undergoes non-linear saturation. It subsequently reaches a statistical equilibrium that is maintained over the later evolution of the simulation. The model is thus considered to be relaxed although it has not yet reached a complete thermal relaxation. This point might however not be so important since the initial state of the simulation is directly given by a thermally relaxed evolutionary model. The convective pattern that develops is pre-

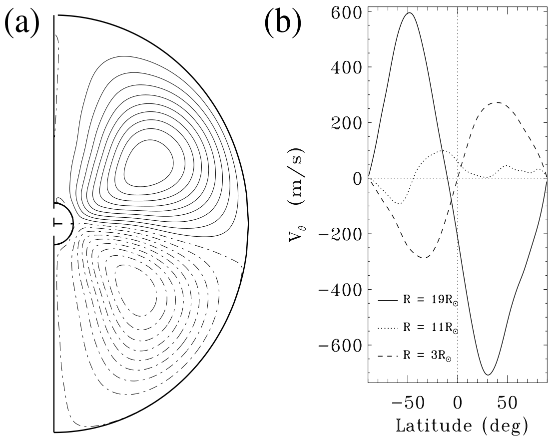

**Fig. 2** (*a*) Meridional circulation averaged over longitude and time represented as streamlines of mass flux. Solid and dashed contours denote counterclockwise and clockwise circulation respectively. (*b*) Mean velocity component $v_\theta$ as a function of latitude at the three indicated depths. Positive values indicate a flow directed from north to south.

sented after 4200 days of evolution in Fig. 1 at three different depths: the top edge (19 $\mathrm{R}_\odot$), the middle (11 $\mathrm{R}_\odot$) and the bottom (3 $\mathrm{R}_\odot$) of the computational domain. The turbulent convection achieved in the simulation is characterized by large warm upflows surrounded by narrower cool downflow lines covering the top edge of the domain. Deeper, the asymmetry between up and downflows is reduced. The correlation between radial velocity and temperature appears clearly in these maps resulting in an outward transport of heat. In order to actually transport the large luminosity of the star ($L \sim 400\ \mathrm{L}_\odot$), the velocity and temperature fluctuations are very large, up to 3000 $\mathrm{m\,s^{-1}}$ and 400 K, respectively. These variations are one order of magnitude larger than those found in the simulations of the solar convective envelope, where the radial velocity and temperature fluctuations do not exceed a few 100 $\mathrm{m\,s^{-1}}$ and 10 K respectively below $r = 0.96\ \mathrm{R}_\odot$. The convective luminosity is found to be larger in the bulk of the domain than the stellar luminosity, since it must compensate a negative kinetic energy luminosity that can represent up to 70% of the total luminosity. This negative kinetic energy flux results from the strong asymmetry between upflows and downflows in the bulk of the domain. This is an important result that contradicts the assumption of the mixing length theory, that assumes that the total and convective luminosity are equal.

The convection is found to posses a significant amount of vorticity as illustrated in Fig.1 by the enstrophy map. It is cyclonic (counterclockwise in the northern hemisphere), and mainly concentrates around the strongest downflows. Similarly the helicity of the vortices is found to be antisymmetric with respect to the equator, with the vortices in the northern hemisphere having preferentially negative helicity. However the overall kinetic helicity is the same in both hemispheres.

In the bulk of our simulation, the kinetic energy density (KE) is dominated by convection, which represents 57% of the total, while the kinetic energy of differential rotation and of the meridional circulation represent respectively 34% and 9%. Contrary to the solar case, where the contribution of meridional circulation to the averaged total KE is less than 1%, the meridional circulation here is not negligible in the total energy budget. The meridional circulation pattern achieved in the simulation consists of one poleward cell per hemisphere. This pattern, presented in panel (a) of Fig. 2, is very stable over the simulation, in particular, no inversions of the meridional circulation are observed, and the total number of cells is always two. The typical amplitudes of these large-scale circulations are about 600 $\mathrm{m\,s^{-1}}$ at the top of the computational domain and 200 $\mathrm{m\,s^{-1}}$ at the bottom, much larger than what is found in the Sun (about 20 $\mathrm{m\,s^{-1}}$ at the solar surface). The turnover time for the meridional circulation is about 3.5 years, to be compared to the 150 days for the characteristic convective overturning time.

## 4 Angular momentum redistribution in the shell

In this extended convective envelope, our choice of stress-free velocity boundaries ensures that no torque is applied to the system, and that angular momentum is conserved in the convective shell. In turbulent convection zones, the angular momentum is redistributed by various physical processes, which are in turn, viscous diffusion, Reynolds stresses, and mean large scale circulations. We may identify the contribution to the transport of angular momentum of each process by considering the mean radial ($\mathcal{F}_r$) and latitudinal ($\mathcal{F}_\theta$) angular momentum fluxes. The expressions of these fluxes are given in Brun & Toomre (2002). Figure 3a,b presents these integrated averaged fluxes along co-latitude and radius respectively. Let's first note the good quality of the angular momentum balance characterized by the solid lines, that is very close to zero (e.g. no net flux). In the radial direction, meridional circulation acts to transport the angular momentum outward. The Reynolds stresses, that are associated with velocity correlations $\langle v_r' v_\phi' \rangle$, act so as to balance the meridional circulation by transporting the angular momentum inward. The transport by viscous diffusion occurs down the gradient of angular velocity, opposite to the Reynolds stresses, and is negligible. In the latitudinal direction, the viscous forces transport angular momentum in the same direction as Reynolds stresses, their averaged integrated fluxes being negative in the southern hemisphere and positive in the northern hemisphere. Here again, the meridional circulation flux compensates that of the Reynolds stresses, with a poleward transport in the northern hemisphere.

The redistribution of angular momentum in this convective shell results in the evolution from an initial solid-body rotation regime in the simulation to a strong differentially rotating regime both in latitude and radius. This strong dif-

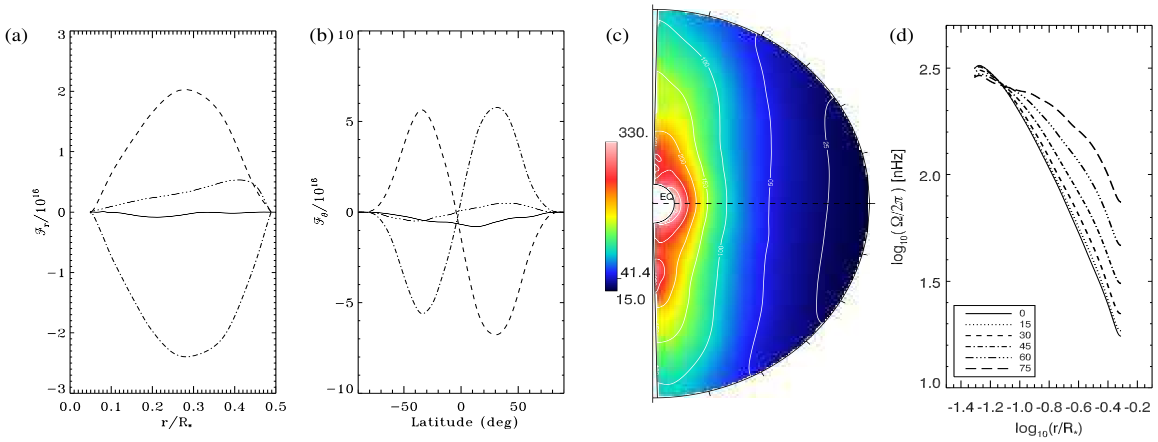

**Fig. 3** (online colour at: www.an-journal.org) Time average of the latitudinal line integral of the angular momentum flux $\mathcal{F}_r$ (*a*) and of the radial line integral of the angular momentum flux $\mathcal{F}_\theta$ (*b*) for the model. The fluxes have been decomposed into their viscous (dashed-triple-dotted), Reynolds stresses (dotted-dashed) and meridional circulation (dashed) components. The solid curves represent the total fluxes. The positive values are for radial flux directed outward, and latitudinal fluxes directed from north to south. The fluxes have been averaged over the entire simulation. (*c*) Temporal and longitudinal average of the angular velocity profile achieved in the simulation over 2300 days. The reference frame rotation rate $\Omega/2\pi$ is 41.4 nHz. Radial profiles are plotted in (*d*) for selected latitudes.

ferential rotation is shown in Fig. 3c,d. The relative latitudinal contrast of angular velocity $\Delta\Omega/\Omega_0$ between $0^o$ and $60^o$ near the top of the domain is close to 60%. As can be seen on the contour plots of this figure (panel c), there is a strong alignment of $\Omega$ contours with the rotation axis at almost all latitudes, with $\Omega$ being almost constant on cylinders. We may attribute this cylindrical rotation to the mild degree of turbulence of the present simulation or to the absence of a strong latitudinal entropy contrast (Miesch et al. 2006). However the large extension of the convective envelope makes it difficult to generalize results obtained in solar simulations to RGB stars. The inner prograde regions rotate as a whole, faster than the outer retrograde ones, and the resulting rotation is anti-solar, the poles rotating faster than the equatorial zone. This is could be due to the moderate convective Rossby number of 0.752 achieved in the simulation (Browning et al. 2004), or expressed differently to the weak influence of the Coriolis force on the rising convective blobs. Overall the large differential rotation achieved in the simulation is due to the competition of two processes: Reynolds stresses and large scale axisymmetric meridional flows.

## 5 Conclusion

This first simulation has now been evolved over more than 4000 days, and presents a statistical equilibrium in terms of the rotation profile, the meridional circulation and the energy budget achieved. The convection patterns present a large asymmetry between upflows and downflows, which results in an enthalpy flux representing up to 170% of the total flux in the bulk of the domain. This result is the first important one, since it contradicts one of the hypothesis of the mixing length theory, that assumes that the convective flux is the sum of enthalpy and radiative flux, neglecting the kinetic energy flux. This should be somehow further investigated within the 1-D stellar evolution models, since it could affect the evolution of stars. Another important result concerns the angular momentum redistribution in the simulation, even though it can not be taken as a firm conclusion at the moment. We observe the development and maintenance of a large cylindrical differential rotation within the inner part of the convective envelope of our red giant (see also Steffen & Freytag in the same volume). This regime is opposed to the assumption of solid body rotation assumed in stellar evolution modeling. It could be of great importance for the global redistribution of angular momentum and chemical species in the underlying radiative region. We shall thus explore in more details the parameter space of our simulation in order to be able to propose a more realistic prescription for the angular momentum distribution within the convective envelope of red giant stars for 1-D stellar evolution.

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