Mixing Below the Solar Convective Zone

A. S. Brun[1,2] and S. Turck-Chièze

[1]*DSM/DAPNIA/Service d'Astrophysique, CEA Saclay, 91191 Gif-sur-Yvette Cedex, France*

J. P. Zahn

[2]*Département d'Astrophysique Stellaire et Galactique, Observatoire de Paris, Section Meudon, 92195 Meudon, France*

Abstract. Helioseismology has shown that the transition from differential rotation in the convection zone to almost uniform rotation in the radiation interior occurs in a shallow layer, the tachocline. This layer is presumably the seat of a large scale circulation and of turbulent motions. Starting from updated solar models (Brun, Turck-Chièze & Morel 1998), we use a physical description of the tachocline proposed by Spiegel & Zahn (1992) to calculate the mixing within this layer, where the thickness is deduced from helioseismic inversion. As a result of this mixing we note a significant improvement for the surface chemical abundance of light element such as ^{4}He, ^{7}Li and ^{9}Be, and a reduction of the sound speed square difference, between the seismic Sun and the models.

1. Introduction

For most purposes, the Sun can be assumed to be in hydrostatic and thermal equilibrium, neglecting the effects of rotation and magnetic field. However models built with these simplifying assumptions do not agree with the helioseismic data, in particular with those obtained by the satellite SOHO, and it appears that macroscopic mixing processes must be taken into account not only in the convection zone, but also in the radiative interior.

In this study, we deal with the modeling of the thin transition layer between the convective and the radiative zone. Following the description of Spiegel & Zahn (1992) of the motions in this so-called tachocline and that of Chaboyer & Zahn (1992) for the chemical evolution, we build several solar models, which we compare to the helioseismic and surface abundance observations.

2. Mixing in the solar tachocline : physical description

Turbulent mixing may be introduced in solar models by adding a turbulent diffusivity D_T in the equation for the time evolution of the concentration of chemical species. To establish this coefficient for the tachocline, we use the description by Spiegel & Zahn (1992), who have given a physical interpretation of this layer by invoking anisotropic turbulence, with much stronger viscous transport in the

horizontal than in the vertical direction. Such turbulence reduces the differential rotation and therefore inhibits the spread of the layer deep inside the radiative zone. This anisotropic diffusion will also interfere with the advective transport of chemicals. Chaboyer & Zahn (1992) have shown that the result is a diffusive transport in the vertical direction. Using their result, Brun, Turck-Chièze & Zahn (1998) derived the following expression for the turbulent diffusivity:

$$D_T(r) = \frac{1}{180}\frac{1}{4}\left(\frac{8}{3}\right)^2 \nu_H \left(\frac{d}{r_{bcz}}\right)^2 \mu_4^6 Q_4^2 \exp(-2\zeta) cos^2(\zeta) + \text{higher order terms} \quad (1)$$

with $Q_4 = \tilde{\Omega}_4/\Omega$, $\tilde{\Omega}_4$ characterizing the differential rotation rate, $\mu_4 = 4.933$, $\zeta = \mu_n(r_{bcz} - r)/d$, $d = r_{bcz}(2\Omega/N)^{1/2}(4K/\nu_H)^{1/4}$ a variable related to the tachocline thickness h (e.g $h \sim d/2$), r_{bcz} the radius, Ω the angular velocity, N the Brunt-Väisälä frequency and $K = \chi/\rho c_p$ the radiative diffusivity at the base of the convective zone. The horizontal component of the turbulent diffusivity is assumed to be equal to the horizontal viscosity ν_H. In our solar models, we shall treat h (hence d) as an adjustable parameter, chosen to agree with the helioseismic inversion of the tachocline thickness $h \leq 0.05 R_\odot$ (Corbard et al. 1999). With the latitudinal dependence of Ω borrowed from Thompson et al. (1996), $\Omega_{bcz}/2\pi = 456 - 72x^2 - 42x^4$ nHz, the numerical coefficient Q_4 is -1.707×10^{-2} and the ratio between the rotation in the deep radiative zone and the equatorial rate at the base of the convection zone is $\Omega/\Omega_0 = 0.9104$. The prediction by Gough & Sekii (1997), who consider instead the magnetic stresses, is ~ 0.96; presently, the seismic observations suggest a rotational ratio of 0.94 ± 0.01 (Corbard et al. 1999), which is intermediate between these two theoretical estimates.

An analysis of the dependence of our d and D_T with the global and differential rotation rates yields:

$$D_T \propto \Omega \nu_H^{1/2} (\hat{\Omega}/\Omega)^2, \qquad d \propto \Omega^{1/2}/\nu_H^{1/4}. \quad (2)$$

Assuming that the turbulent viscosity is proportional to the differential rotation (i.e. $\nu_H \propto \hat{\Omega}$) and introducing the dependence of the differential rotation on rotation observed by Donahue, Saar & Baliunas (1996) ($\hat{\Omega} \propto \Omega^{0.7\pm0.1}$), we finally obtain for $D_T(\Omega)$ and $d(\Omega)$:

$$D_T \propto \Omega^{0.75\pm0.25}, \qquad d \propto \Omega^{(1.3\pm0.1)/4}. \quad (3)$$

We render the mixing in the tachocline time-dependent, through $D_T(\Omega(t))$ and $d(\Omega(t))$, by using the spin-down law $\Omega \propto t^{-1/2}$ which was deduced by Skumanich (1972) from the rotation rate of stellar clusters of different ages.

3. Results

Starting from the reference model of Brun, Turck-Chièze & Morel (1998) built with the CESAM code (Morel 1997), we introduce the coefficient D_T in the diffusion equation of chemical species, and we follow the time evolution of our model from the pre-main sequence until 4.55 Gyr. The results are shown in the

Table 1 and Figure 1. We use a tachocline thickness h of 0.05 or 0.025 $R_\odot$, a N of 100 or 20 μHz and a Ω of 0.415 μHz. Our standard model (Brun, Turck-Chièze & Morel 1998), has a surface abundance of helium of 0.2432 in mass, which is a bit too low if we compare with the Basu & Antia (1995) value for the OPAL equation of state (Rogers, Swenson & Iglesias 1996), $^4\mathrm{He}_s = 0.249 \pm 0.003$. When introducing our diffusive coefficient, we mix back helium in the convection zone, inhibiting the microscopic diffusion up to 25% and leading to a $^4\mathrm{He}_s = 0.2481$ (c.f Table 1, models D_{TA} and D_{TB}). As expected, the composition profile is smoother and flattens over the distance h below the convective zone (see Brun, Turck-Chièze & Zahn 1998).

Table 1. Surface abundances of ^{4}He and heavy elements Z and abundance ratio initial/surface for ^{7}Li and ^{9}Be from observations and for solar models at the solar age (subscript t for time dependent models)

	Obs	Ref	D_{TA}	D_{TB}	D_{TAt}	D_{TBt}	D_{TBtz}	D_{TCt}
d $(r/R_\odot)$	≤ 0.1	-	0.1	0.1	0.1	0.1	0.1	0.05
N (μHz)	-	-	100	20	100	20	20	20
$^4\mathrm{He}_s$	0.249±0.003	0.2432	0.2456	0.2481	0.2459	0.2486	0.2519	0.2471
$(\mathrm{Z/X})_s$	0.0245±0.002	0.0245	0.0245	0.0245	0.0245	0.0245	0.0255	0.0245
$^7\mathrm{Li}_0/^7\mathrm{Li}_s$	∼100	∼5	∼6	∼20	∼9	∼91	∼132	∼79
$^9\mathrm{Be}_0/^9\mathrm{Be}_s$	1.10±0.03	1.11	1.092	1.085	1.092	1.126	1.14	1.093

For the models D_{TAt}, D_{TBt} and D_{TCt}, we calculate the evolution with the time-dependent diffusivity of equation (3). Recent abundance determinations (Grevesse, Noels & Sauval 1996 and Balachandran & Bell 1997) have shown that the solar ^{7}Li is depleted by a factor ~ 100 and that it is not the case for ^{9}Be. These two light chemical elements are really sensitive to mixing processes occuring in the stars because their nuclear burning temperatures are very low (resp. 2.5 10^6 and 3.2 10^6 K) (Baglin & Lebreton 1990). These new observational constraints can only be satisfied if those chemical species are mixed in a rather thin layer below the convective zone, which is the case for the tachocline mixing considered here. In Table 1 we give the initial over present ratio of ^{7}Li and ^{9}Be. With the coefficients D_{TB} more ^{7}Li is burned than with D_{TA}, and we also see that the time dependence improves the ^{4}He surface abundance as well as the ^{7}Li depletion, where a value of ~ 100 is obtained without destroying ^{9}Be, in agreement with the observations.

In Figure 1 (left) we show the lithium depletion occurring during the Sun's evolution for the different models presented, plus a model without any diffusion (see also Vauclair & Richard 1998). Clearly, only the diffusive models including mixing in the tachocline show a substantial depletion during main sequence evolution, in agreement with the observations (superimposed with their inherent dispersion on the theoretical curves). Note that the strong time dependent mixing with $N = 20$ (models D_{TBt}, D_{TCt} and D_{TBtz}) can account for all the ^{7}Li depletion which is observed, although it may be somewhat too strong in the early phases.

When comparing the sound speed profiles with the helioseismic inversion (Fig 1, right), we see that the conspicuous bump is practically erased by this process. The model D_{TBtz} with an initial heavy elements composition $\mathrm{Z}_0 = \mathrm{Z}_0^{ref} = 0.01957$ seems even in better agreement with the helioseismic data, keeping a Z/X surface abundance within the observational error bars (c.f Table 1).

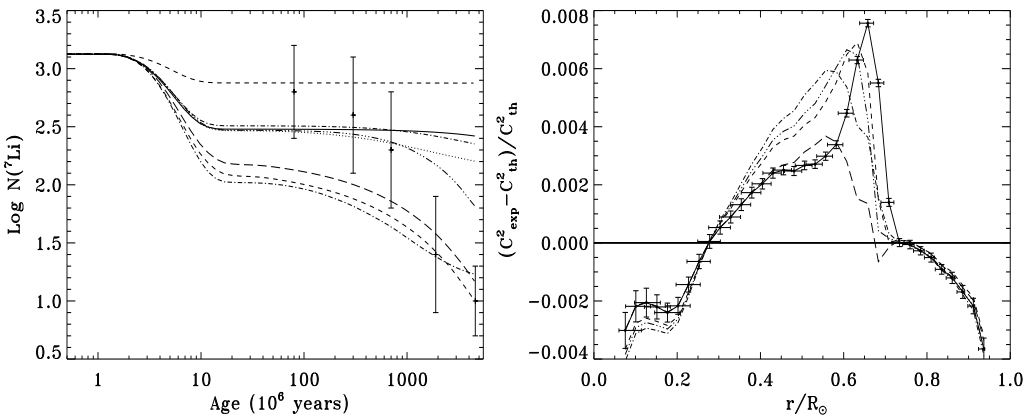

Figure 1. (Left) Time dependent depletion of ^{7}Li for several solar models: no microscopic diffusion (- - -), with microscopic diffusion (full line) and with mixing in the tachocline thickness: coefficient D_{TA} (-.-), D_{TB} (-...-), then with time dependent mixing D_{TAt} (...) and D_{TBt} (long dash), D_{TCt} (thick -.-) and D_{TBtz} ($Z_0 = Z_0^{ref} = 0.01957$) (thick - -). We superimposed the open cluster observations (adapted from Vauclair & Richard 1998). (Right) Sound speed square difference between GOLF+MDI data, the reference model (full line) and models including a turbulent term: coef D_{TAt} (- - -), D_{TBt} (-.-), D_{TCt} (-...-) and one model with $Z_0 = Z_0^{ref} = 0.01957$, D_{TBtz} (long dash).

References

Baglin, A. & Lebreton, Y. 1990, Inside the Sun, eds G. Berthomieu & M. Cribier, Kluwer Academic Publishers, Netherlands, 437.

Balachandran, S. & Bell, R. A. 1997, Amer. Astron. Soc. Meeting., 191, #74.08.

Basu, S. & Antia, H. M. 1995, MNRAS, 276, 1402.

Brun, A. S., Turck-Chièze, S. & Morel, P. 1998, ApJ, 506, 913.

Brun, A. S., Turck-Chièze, S. & Zahn, J. P. 1998, in Structure and Dynamics of the Interior of the Sun and Sun-like Stars, eds S. G. Korzennik & A. Wilson, (ESA: Noordwijk), ESA SP-418, Vol 1, 439.

Chaboyer, B. & Zahn, J. P. 1992, A&A, 253, 173.

Corbard, T., Blanc-Féraud, L., Berthomieu, G. & Provost, J. 1999, to appear in A&A.

Donahue, R. A., Saar, S. H. & Baliunas, S. L. 1996, ApJ, 466, 384.

Gough, D. O. & Sekii, T. 1997, IAU 181 Sounding Solar and Stellar Interior (poster volume), eds J. Provost & F. X. Schmider, Observatoire de la Côte d'Azur, 93.

Grevesse, N., Noels, A., & Sauval, A. J. 1996, in Cosmic Abundances, ed. S. S. Holt & G. Sonneborn, ASP Conference Series, 117.

Morel, P. 1997, A&A Sup., 124, 597.

Rogers, F. J., Swenson, J. & Iglesias, C. 1996, ApJ, 456, 902.

Skumanich, A. 1972, ApJ, 171, 565.

Spiegel, E. A. & Zahn, J. P. 1992, A&A, 265, 106.

Vauclair, S. & Richard, O. 1998, in Structure and Dynamics of the Interior of the Sun and Sun-like Stars, eds S. G. Korzennik & A. Wilson, (ESA: Noordwijk), ESA SP-418, Vol 1, 427.