Influence of the Tachocline on Solar Evolution

A. S. Brun[1] and J.-P. Zahn

Département d'Astrophysique Stellaire et Galactique, Observatoire de Paris, Section Meudon, 92195 Meudon, France

Abstract. Recently helioseismic observations have revealed the presence of a shear layer at the base of the convective zone related to the transition from differential rotation in the convection zone to almost uniform rotation in the radiative interior, the tachocline. This layer extents only over a few percent of the solar radius at the present day and no definitive explanations have been given for this thiness. Following Spiegel & Zahn (1992), who invokes anisotropic turbulence to stop the spread of the tachocline deeper in the radiative zone as the Sun evolves, we give some justifications for their hypothesis by taking into account recent results on rotating shear instability (Richard & Zahn 1999). Then we study the impact of the macroscopic motions present in this layer on the Sun's structure and evolution by introducing a macroscopic diffusivity D_T in updated solar models. We find that a time dependent treatment of the tachocline improves significantly the agreement between computed and observed surface chemical species, such as the ^{7}Li and modify the internal structure of the Sun (Brun, Turck-Chièze & Zahn 1999).

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1. Introduction

The presence of a shear layer connecting the differential rotation of the convective zone to the solid rotation of the radiative zone is now well established by helioseismic inversions (Figure 1 and Corbard et al. 1999). Both its location and width are more and more constrained and seem to be respectively 0.691 ± 0.004 $R_\odot$ and less than 0.05 $R_\odot$. Today, there are several hydrodynamical or MHD descriptions of this shear layer and its extension but none is definitive (Spiegel & Zahn 1992 (hereafter; SZ92), Gough & McIntyre 1998 and Gilman & Fox 1997). In these models the motions in the tachocline are either turbulent or laminar, involve magnetic field or are purely hydrodynamical. In this paper we shall discuss the basic ideas supporting the dynamical description of this transition layer. We first recall in section 2, the physical processes acting in this layer and summarize the different approaches with an emphasis on Spiegel & Zahn's

[1]Now at JILA, University of Colorado, Boulder, CO 80309-0440, USA.

description invoking a non-linear anisotropic turbulence. Then in section 3, using the prescription of SZ92 for the amplitude of the vertical velocity in the tachocline and Chaboyer & Zahn (1992) for the chemical mixing and evolution, we build solar models including a macroscopic diffusivity D_T, which are compared to the most recent helioseismic data and surface abundance observations for ^{7}Li, ^{9}Be and ^{3}He/^{4}He ratio. Finally we summarize our results and conclude in section 4.

2. The solar tachocline

The transition layer between the convective and radiative zones (Figure 1) plays a crucial role in our understanding of stars such as the Sun because it involves simultaneously several physical features, such as:

- the strong shear associated with the transition from differential to solid rotation, which may generate turbulence,
- a turbulent interface with the convection zone above, which may produce internal waves (Press 1981, Schatzman 1993),
- the possible presence of magnetic field, of fossile origin (Gough & McIntyre 1998), or linked with the 11 years cycle (Chouduri, Schüssler & Dikpati 1997),
- the proximity of the thermonuclear burning zone of lithium 7 and beryllium 9.

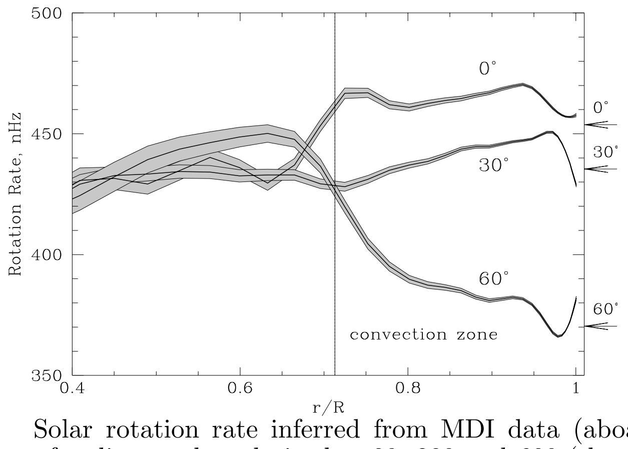

Figure 1. Solar rotation rate inferred from MDI data (aboard SOHO) as a function of radius at three latitudes, 0^o, 30^o and 60^o (the vertical line represents the base of the convection zone) (Kosovishev et al 1997).

In this section we shall concentrate on some specific points concerning the tachocline and we will not deal with the full complexity of this transition layer. The presence of a latitudinal differential rotation at the base of the convection zone induces a latitudinal temperature gradient $\Omega(\theta) \rightarrow \nabla T(\theta)$. Without any limitating process this temperature gradient will diffuse inwards, on a thermal diffusion time scale, enforcing differential rotation deep into the radiative interior of the Sun. But as we already stated, helioseismic observations indicate a uniform rotation profile in the radiation zone, and thus we have to find which processes could hinder this diffusion.

There are different possibilities and the first of all is the stable stratification of the radiation zone. This effect would indeed slow down the spread of the tachocline, but in spite of that, the layer would still extend to one third of the radius in the present Sun, as estimated by SZ92, which is far too much. Thus another process must be invoked to explain the observed thinness of the tachocline. One possibility is the presence of magnetic field in the radiation zone, as advocated by Gough and McIntyre (1998). Their model is promising, but it has not been worked out in detail yet: it remains to be seen how the poloidal field threads into the convection zone, and avoids imposing differential rotation throughout the radiative interior.

Another possibility has been suggested by SZ92. If the latitudinal shear is unstable, it could generate anisotropic turbulence which would tend to reduce the differential rotation, and hence prevent the spread of the tachocline. It is not clear whether the solar tachocline is linearly unstable: when one applies the criterion derived by Watson (1981) to an angular velocity profile of the type $\Omega \propto 1 - \alpha_1 \sin^2 \lambda$, where λ is the latitude, linear instability requires $\alpha_1 > 0.29$, which is larger than the solar value $\alpha_1 \approx 0.25$. However it appears that a law of the type $\Omega \propto 1 - \alpha_1 \sin^2 \lambda - \alpha_2 \sin^4 \lambda$, closer to the latitudinal dependence drawn from helioseismic inversions (see Fig 1) would be more sensitive to such instability (Garaud & Gough 1999). Furthermore, a toroidal magnetic field of sufficient strength would also act to destabilize the flow, as shown by Gilman and Fox (1997). In any case, the Reynolds number is so high that such a differential rotation would be liable to finite amplitude instability, as can be infered from laboratory experiments (Richard & Zahn 1999).

It has been argued by Gough and McIntyre (1998) that turbulence does not necessarily reduce the shear of differential rotation: in the Earth's atmosphere the transport of angular momentum would even imply a negative viscosity. But there the transport is achieved mainly through Rossby waves, and it is not clear whether this can be applied to the solar tachocline.

Admittedly this important issue is still a matter of debate, and it may be settled only by comparing the models' predictions with the observed properties of the Sun. This is why we have chosen to draw all observable consequences from the model which has been worked out in sufficient detail to allow such a test, namely the turbulent tachocline proposed by Spiegel and Zahn (1992). We will show that the mixing induced in this model improves both the sound speed profile (reduction of the peak below the convection zone) and the surface light element abundances, confirming the need of introducing macroscopic processes in solar models (Zahn 1998; Vauclair & Richard 1998; Brun, Turck-Chièze & Zahn 1999).

3. Mixing in the solar tachocline : physical description

Macroscopic mixing may be treated in solar models by adding an effective diffusivity D_T in the equation for the time evolution of the concentration of chemical species. To establish this coefficient for the tachocline, we use the description by SZ92, where anisotropic turbulence is responsible for stopping the spread of the layer. This anisotropic diffusion will also interfere with the advective transport of chemicals. Chaboyer & Zahn (1992) have shown that the result is a diffusive transport in the vertical direction. Using their result, Brun, Turck-Chièze & Zahn (1998) derived the following expression for the macroscopic diffusivity:

$$D_T(r) = \frac{4}{405}\left(\frac{8}{3}\right)^2 \nu_H \left(\frac{d}{r_{bcz}}\right)^2 \mu_4^6\, Q_4^2 \exp(-2\zeta)\cos^2(\zeta) + \text{higher order terms} \tag{1}$$

with $Q_4 = \tilde{\Omega}_4/\Omega$, $\tilde{\Omega}_4$ characterizing the differential rotation rate, $\mu_4 = 4.933$, $\zeta = \mu_4(r_{bcz} - r)/d$ a non-dimensional depth,

$$d = r_{bcz}(2\Omega/N)^{1/2}(4K/\nu_H)^{1/4} \tag{2}$$

a length related to the tachocline thickness h (e.g $h \sim d/2$), r_{bcz} the radius, Ω the angular velocity and $K = \chi/\rho c_p$ the radiative diffusivity at the base of the convective zone. The horizontal component of the macroscopic diffusivity D_H is assumed to be equal to the horizontal viscosity ν_H. In our solar models, we shall treat h (hence d) as an adjustable parameter, chosen to agree with the helioseismic determination of the tachocline thickness $h \leq 0.05 R_\odot$ (Corbard et al. 1999). With the latitudinal dependence of the angular velocity at the base of the convection zone deduced from Thompson et al. (1996), $\Omega_{bcz}/2\pi = 456 - 72x^2 - 42x^4$ nHz (with $x = \sin\lambda$), we have reestimated the coefficient $Q_4 = -1.707 \times 10^{-2}$, as well as the ratio between the rotation in the deep radiative zone and the equatorial rate $\Omega/\Omega_0 = 0.9104$. The prediction by Gough & Sekii (1997) for the latter, who consider instead the magnetic stresses, is ~ 0.96; presently, the seismic observations suggest a rotational ratio of 0.94 ± 0.01 (Corbard et al. 1999), which is intermediate between these two theoretical estimates.

An analysis of the dependence of our d and D_T with the global and differential rotation rates yields

$$D_T \propto \nu_H \left(\frac{d}{r_{bzc}}\right)^2 Q_i^2 \propto \Omega \nu_H^{1/2} (\hat{\Omega}/\Omega)^2, \qquad d \propto \Omega^{1/2}/\nu_H^{1/4}. \tag{3}$$

where we have used equations (1) and (2). Assuming that the turbulent viscosity is proportional to the differential rotation (i.e. $\nu_H \propto \hat{\Omega}$), as suggested by the laboratory experiments (Richard & Zahn 1999), and introducing the dependence of the differential rotation on rotation observed by Donahue, Saar & Baliunas (1996) ($\hat{\Omega} \propto \Omega^{0.7\pm0.1}$), we finally obtain the following scalings

$$D_T \propto \Omega^{0.75\pm0.25}, \qquad d \propto \Omega^{(1.3\mp0.1)/4}. \tag{4}$$

We conclude that the tachocline mixing was stronger in the past both because that layer was thicker and because the diffusivity was larger. We render the mixing in the tachocline time dependent, through $D_T(\Omega(t))$ and $d(\Omega(t))$, by using the spin-down law $\Omega \propto t^{-1/2}$ which was deduced by Skumanich (1972) from the rotation rate of stellar clusters of different ages.

4. Results

Starting from the reference model of Brun, Turck-Chièze & Morel (1998) built with the CESAM code (Morel 1997), we introduce for this study the nuclear reaction ^{7}Li(p,α)^{4}He proposed by Engstler et al. (1992) and the coefficient D_T (equation 1) in the diffusion equation of chemical species, and we follow the time evolution of the solar model from the pre-main sequence until 4.6 Gyr. The results are shown in the Table 1 and Figures 2-5 (see Brun, Turck-Chièze & Zahn 1999 for a more detailed discussion). We use a tachocline thickness h of 0.05 or 0.025 $R_\odot$, N of 100 or 25 μHz and Ω of 0.415 μHz. Our standard model (Brun, Turck-Chièze & Morel 1998) has a surface abundance for helium of 0.2427 in mass, corresponding to an ^{4}He difffusion of 10.8%. This value of Y_s is a bit too low if we compare with the Basu & Antia (1995) value for the OPAL equation of state (Rogers, Swenson & Iglesias 1996), $Y_s = 0.249 \pm 0.003$.

Table 1. Surface abundance variation of ^{3}He/^{4}He during the last 3 Gyr, surface abundances of ^{4}He and heavy elements Z, and abundance ratio initial/surface for ^{7}Li and ^{9}Be from observations and for solar models at the solar age (subscripts t for time dependent models, tz for time dependent model with $Z_0 = Z_0^{ref} = 0.01959$

	Obs	Ref	A	B	A_t	B_t	B_{tz}	C_t
d $(r/R_\odot)$	≤ 0.1	-	0.1	0.1	0.1	0.1	0.1	0.05
N (μHz)	-	-	100	25	100	25	25	25
$(^3\mathrm{He}/^4\mathrm{He})_s$	max 10%	2.28%	2.14%	2.01%	2.11%	2.0%	2.02%	2.07%
$^4\mathrm{He}_s$	0.249±0.003	0.2427	0.2452	0.2473	0.2455	0.2477	0.2509	0.2464
$(\mathrm{Z/X})_s$	0.0245±0.002	0.0245	0.0245	0.0245	0.0245	0.0245	0.0255	0.0245
$^7\mathrm{Li}_0/^7\mathrm{Li}_s$	∼100	∼6	∼8	∼22	∼12	∼91	∼134	∼89
$^9\mathrm{Be}_0/^9\mathrm{Be}_s$	1.10±0.03	1.115	1.093	1.086	1.093	1.118	1.125	1.092

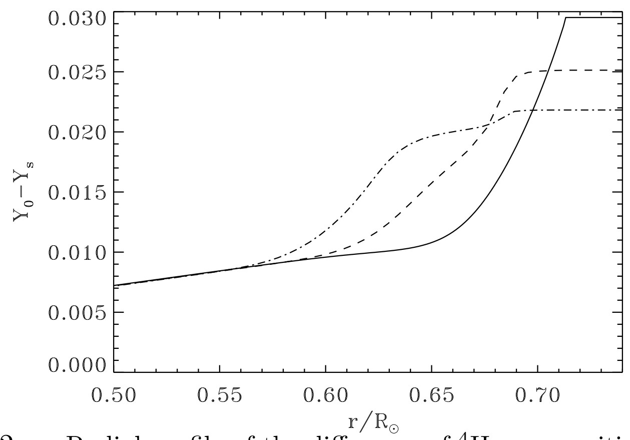

Figure 2. Radial profile of the difference of ^{4}He composition between the initial and present values for the reference solar model including only microscopic diffusion (solid line) and solar models where we add a macroscopic mixing due to the presence of the tachocline: coefficient A (dash) and B (dash dot) (see Table 1 for the model characteristics).

When introducing our diffusive coefficient, we mix helium back into the convection zone, inhibiting the microscopic diffusion up to 25% and producing a photospheric $^4\mathrm{He}_s = 0.2473$ (cf. Table 1, models A and B). As expected,

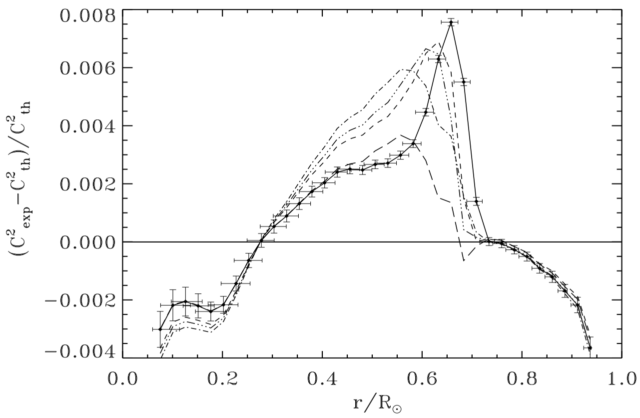

Figure 3. Squared sound speed difference between GOLF+MDI data and the reference model (solid line) or models including a turbulent term: coef A_t ($d = 0.1$, $N = 100$) (dash), B_t ($d = 0.1$, $N = 25$) (dash dot), C_t ($d = 0.05$, $N = 25$) (dash three dots), and one model with B_t and $Z_0 = Z_0^{ref} = 0.01959$ (i.e B_{tz})(long dash).

the composition profile is smoother and flattens over the distance h below the convective zone (see Figure 2).

The effect on the sound speed is displayed in Fig. 3. When the macroscopic transport is neglected, the squared sound speed difference reveals a peak just below the convection zone, coinciding with the tachocline (continuous line). Macroscopic diffusion acts to reduce this peak, but when one recalibrates the model such as to yield the present abundance of heavy elements (Z/X=0.0245), the effect is rather minor. On the other hand, if the heavy element is let free to adjust, within the observational uncertainties, the peak is completley removed, leaving only a broad bump culminating at 0.6 $R_\odot$, which presumably is due to another cause (long dashed line).

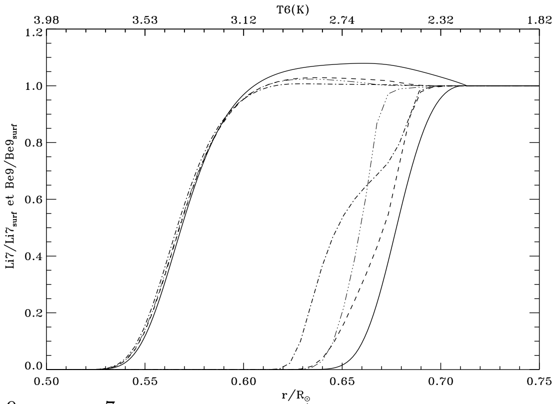

Figure 4. ^{9}Be and ^{7}Li radial profile (approximately superimposed the temperature scale) for several models. Respectively: reference (solid line), coefficient A_t (dash), coefficient B_t (dash dot) and coefficient C_t (dash three dots).

The two light elements ^{7}Li and ^{9}Be are extremely sensitive to mixing processes occuring in stars because their nuclear burning temperatures are rather low (resp. 2.5 10^6 and 3.2 10^6 K) (Baglin & Lebreton 1990). The new observational constraints can only be satisfied if those chemical species are mixed in a rather thin layer below the convective zone, in order to preserve ^{9}Be, which is very little depleted according to Balachandran and Bell (1998). This is the case with our tachocline model. However, if the mixing had proceeded in the past at the same rate as in the present Sun, ^{7}Li would have been depleted only by a factor ~ 4, which is insufficient to account for the photospheric lithium abundance (Cayrel 1998 and references therein). But the thickness of the tachocline and the strength of mixing have been larger in the past, when the Sun was rotating faster. This effect is included in the models labeled with the index t (as B_t), whose evolution was calculated with the time-dependent diffusivity of equation (4).

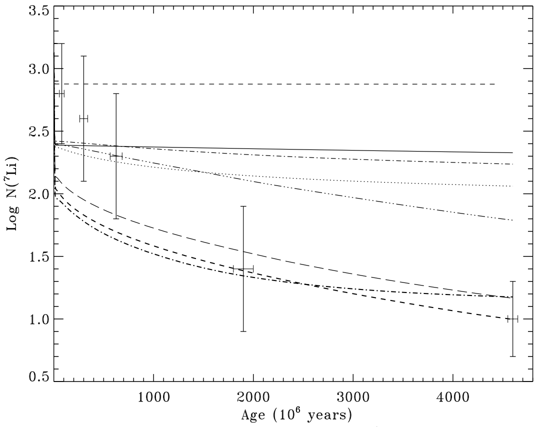

Figure 5. Time dependent depletion of ^{7}Li for several solar models: no microscopic diffusion (- - -), with microscopic diffusion (solid line) and with mixing in the tachocline thickness: coefficient A (-.-), B (-...-), then with time dependent mixing A_t (...) and B_t (long dash), C_t (thick -.-) and B_{tz} ($Z_0 = Z_0^{ref} = 0.01959$) (thick - -). We superimposed on the theoretical curves the open cluster observations (adapted from Vauclair & Richard 1998 and cluster age uncertainties deduced from Lebreton et al. 1997)

In Table 1 we give the initial over present ratio of ^{7}Li and ^{9}Be and show in Figure 4 the radial profile of ^{7}Li and ^{9}Be normalized to the surface abundance. We clearly see that the mixing process modifies the distribution of lithium but not that of beryllium (exception being the flat plateau for the mixed models in comparison with the “pure” diffusive one). With the coefficients B more ^{7}Li is burned than with A, and we also see that the time dependence (model with index t) improves the ^{4}He surface abundance as well as the ^{7}Li depletion, where a value of ~ 100 is obtained without destroying ^{9}Be or increasing too much the ^{3}He/^{4}He surface ratio over the past 3 Gyr, as deduced by Geiss & Gloeckler (1998) from meteorites and solar wind abundance measurements (see Table 1).

In Figure 5 we show the lithium depletion occurring during the Sun’s evolution for the different models presented, plus a model without any diffusion. Clearly, only the diffusive models including mixing in the tachocline yield a substantial depletion during main sequence evolution, in agreement with the observations (superimposed with their inherent dispersion on the theoretical curves). Note that the strong time dependent mixing with $N = 25$ (models B_t, C_t and B_{tz}) presents a reasonable value of the solar ^{7}Li depletion (~ 100).

However the lithium depletion during the PMS is probably overestimated due to the crude spin-down law we have adopted. A more detailed analysis of these phases is under study, including metallicity effects and more appropriate angular momentum evolution during this phase.

Our results show the interest to follow together the photospheric abundance of the four elements ^{3}He, ^{4}He, ^{7}Li, ^{9}Be, and to examine their sensitivity to the microscopic as well as the macroscopic processes. This study encourages the introduction of macroscopic processes in stellar evolution models, and demonstrates the crucial role of the thin tachocline layer below the convective zone.

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