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The Helioseismic Constraints on ^{7}Li and ^{9}Be from SOHO

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Abstract. The ^{7}Li surface depletion observed in stars like the Sun is one of the long standing astrophysical problems. In this paper we suggest a possible explanation for its destruction in the star's radiative interior using up to date seismic data from SOHO. Recently, helioseismology has shown that the transition from differential rotation in the convection zone to almost uniform rotation in the radiative interior occurs in a shallow layer, the tachocline. This layer is presumably the seat of a large scale circulation and of turbulent motions extending the region of mixing below the base of the convection zone. Starting from updated solar models (Brun, Turck-Chièze & Morel 1998), we use the hydrodynamical description of the tachocline proposed by Spiegel & Zahn (1992) to calculate the mixing within this layer, using helioseismic inversion to constrain its extension. As a result of this mixing we obtain a significant improvement for the surface chemical abundance of light elements such as ^{3}He, ^{4}He, ^{7}Li and ^{9}Be, and a reduction of the sound speed square difference, between the seismic Sun and the models (Brun, Turck-Chièze & Zahn 1999).

1. Introduction

For most purposes, the Sun can be assumed to be essentially in hydrostatic and thermal equilibrium, neglecting the effects of rotation and magnetic fields. However models built with these simplifying assumptions do not agree totally with the helioseismic data, in particular with those obtained by the satellite SOHO. It appears that macroscopic mixing processes must be taken into account not only in the convection zone, but also in the radiative interior.

In this study we introduce a new macroscopic process in solar modeling in order to improve our theoretical description of the Sun. We first briefly review the powerful helioseismic tool used to probe the solar internal structure, and then describe the thin transition layer between the convection and the radiation zone by following the hydrodynamical model of the motions in this so-called tachocline of Spiegel & Zahn (1992). Finally, using the prescription of Chaboyer & Zahn (1992) for the chemical evolution, we build several solar models including a macroscopic diffusivity D_T, which we compare to the most recent helioseismic data and surface abundance observations for ^{7}Li, ^{9}Be and ^{3}He/^{4}He ratio.

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2. The helioseismic tool

During the last thirty years major efforts has been carried out by both the observers and the theoreticians to improve the quality and the accuracy of the data and to develop and increase the efficiency of the seismic diagnostic. Now, with the availability of more than 2 years of SOHO data and long time series ground based observations, the helioseismic constraints allow us to go one step further in our understanding of the internal structure of the Sun. In this section, we will briefly review the basis of the helioseismic tool.

The Sun, as a self-graviting sphere of compressible gas, oscillates around its equilibrium state with a period of about 5-min. These oscillations are interpreted as a superposition of waves propagating inside the star (acting as a resonant cavity), and forming standing waves: the eigenmodes of vibration. By projecting these modes onto spherical harmonics Y_l^m, we write any scalar perturbations as (in the case of the pressure $p^{'}$) (Christensen-Dalsgaard and Berthomieu 1991)

$$p^{'}(r,\theta,\varphi,t) = p^{'}(r)Y_l^m(\theta,\varphi)\exp i\omega_{n,l,m}t \quad ,$$

and the vector displacement $\vec{\xi}$ as

$$\vec{\xi}(r,\theta,\varphi,t) = \left(\xi_r(r), \xi_h(r)\frac{\partial}{\partial\theta}, \xi_h(r)\frac{\partial}{\sin\theta\partial\varphi}\right) Y_l^m(\theta,\varphi)\exp i\omega_{n,l,m}t \quad ,$$

where $\xi_h = 1/(\omega^2 r)[p^{'}/\rho + \Phi^{'}]$ is the horizontal displacement, $\Phi^{'}$ the gravitational potential perturbation, $\omega_{n,l,m}$ the eigenfrequency and ρ the gas density. The quantum numbers n, l, m are respectively the radial order (number of nodes along the radius), the degree (measures the total horizontal waves number on the surface by $k_h \sim L/R_\odot$, with $L = \sqrt{l(l+1)}$) and the azimuthal order (number of nodes along the equator). Restricting ourself to adiabatic oscillations within the Cowling approximation ($\Phi^{'}$ neglected) and considering only small radial wavelength compared to $R_\odot$, we reduce the 6th-order system of equations to a second-order wave equation, with the following dispersion relation (Christensen-Dalsgaard and Berthomieu 1991)

$$k_r^2 = \frac{1}{c_s^2}\left[F_l^2\left(\frac{N^2}{\omega_{n,l,m}^2} - 1\right) + \omega_{n,l,m}^2 - \omega_c^2\right] \quad ,$$

where the squared length of the wave vector is written as the sum of a radial and a horizontal component $|\vec{k}|^2 = k_r^2 + k_h^2$, $k_h^2 = F_l^2/c_s^2$ is the horizontal wave number, $F_l^2 = L^2c_s^2/r^2$ the Lamb frequency, $N^2 = g[1/\Gamma_1 d\ln p/dr - d\ln\rho/dr]$ the Brunt-Väisälä frequency, $\omega_c^2 = c_s^2(1-2dH_\rho/dr)/4H_\rho^2$ the cut-off frequency (~ 5.8 mHz), $H_\rho^{-1} = -d\ln\rho/dr$ the density scale height, Γ_1 the adiabatic exponent and $c_s^2 = \Gamma_1 p/\rho$ the sound speed.

The oscillatory solutions of the wave equation define two types of waves, i.e acoustic ones (with $\omega_{n,l,m} > N, F_l$) and gravity ones (with $\omega_{n,l,m} < N, F_l$).

Only the acoustic modes (the so-called 5-min oscillations) have been observed and give us precise information about the solar internal structure. Today,

the research of gravity modes remains one of the big challenge of modern helioseismology. There are two ways to use the seismic data to probe the internal structure of stars:
a) The direct method: comparison of predicted and observed acoustic frequencies.
b) The indirect method using inversion procedures to deduce radial profile of crucial variables as the squared sound speed c_s^2, the density ρ or the adiabatic exponent Γ_1, the rotation and compare them to computed model.

In this paper, we use two variables extracted from GOLF and MDI data (Gabriel et al., 1997, Kosovishev et al. 1997), namely the internal rotation profile (see Figure 1) and the sound speed profile deduced from observations and compared to up to date solar models (Turck-Chièze et al. 1997).

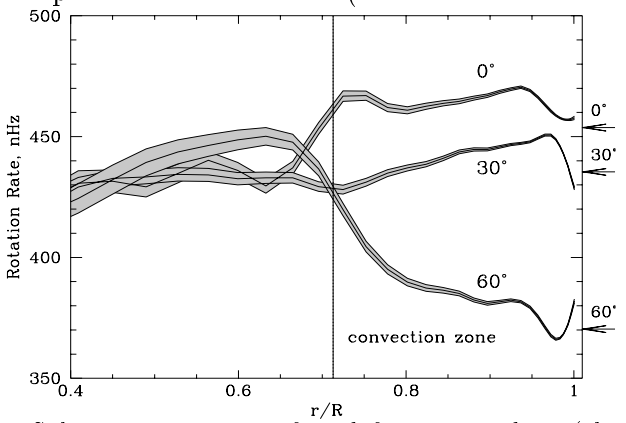

Figure 1. Solar rotation rate inferred from MDI data (aboard SOHO) as a function of radius at three latitudes, 0^o, 30^o and 60^o (the vertical line represents the base of the convection zone) (Kosovishev et al 1997).

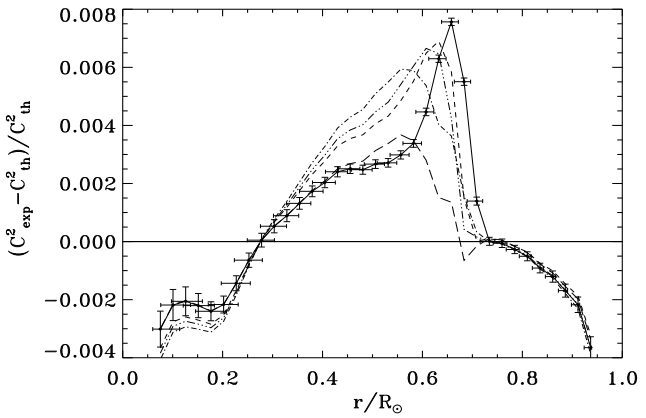

Figure 2. Squared sound speed difference between GOLF+MDI data and the reference model (solid line) or models including a turbulent term: coef D_{TAt} ($d = 0.1$, $N = 100$) (dash), D_{TBt} ($d = 0.1$, $N = 25$) (dash dot), D_{TCt} ($d = 0.05$, $N = 25$) (dash three dots) and one model with D_{TBt} and $Z_0 = Z_0^{ref} = 0.01959$ (long dash).

The characteristic radial squared sound speed difference (see Figure 2 solid line) reveals interesting features: an oscillatory behaviour in the core, a bump between 0.4 - 0.7 $R_\odot$ and a peak below the convection zone ($r \sim 0.68 R_\odot$)

coinciding with the transition between differential rotation in the convective zone to rigid rotation below it (Figure 1). In the next section we show that a first order treatment of the tachocline layer following Spiegel & Zahn (1992) will improve both the sound speed profile (reduction of the peak below the convection zone) and the surface light element abundances, showing the advantage of introducing macroscopic processes in solar models (Brun, Turck-Chièze & Zahn 1999).

3. Mixing in the solar tachocline : physical description

Macroscopic mixing may be introduced in solar models by adding an effective diffusivity D_T in the equation for the time evolution of the concentration of chemical species. To establish this coefficient for the tachocline, we use the description by Spiegel & Zahn (1992), who have given a physical interpretation for this layer by invoking anisotropic turbulence, with much stronger viscous transport in the horizontal than in the vertical direction. Such turbulence reduces the differential rotation and therefore inhibits the spread of the layer deep inside the radiative zone, leading to a stationary state. For a justification of this anisotropy, we refer to Michaud & Zahn (1998). This anisotropic diffusion will also interfere with the advective transport of chemicals. Chaboyer & Zahn (1992) have shown that the result is a diffusive transport in the vertical direction. Using their result, Brun, Turck-Chièze & Zahn (1998) derived the following expression for the macroscopic diffusivity:

$$D_T(r) = \frac{1}{180}\frac{1}{4}\left(\frac{8}{3}\right)^2 \nu_H \left(\frac{d}{r_{bcz}}\right)^2 \mu_4^6\, Q_4^2 \exp(-2\zeta)\cos^2(\zeta) + \text{higher order terms} \tag{1}$$

with $Q_4 = \tilde{\Omega}_4/\Omega$, $\tilde{\Omega}_4$ characterizing the differential rotation rate, $\mu_4 = 4.933$, $\zeta = \mu_n(r_{bcz} - r)/d$, $d = r_{bcz}(2\Omega/N)^{1/2}(4K/\nu_H)^{1/4}$ a variable related to the tachocline thickness h (e.g $h \sim d/2$), r_{bcz} the radius, Ω the angular velocity and $K = \chi/\rho c_p$ the radiative diffusivity at the base of the convective zone. The horizontal component of the macroscopic diffusivity is assumed to be equal to the horizontal viscosity ν_H. In our solar models, we shall treat h (hence d) as an adjustable parameter, chosen to agree with the helioseismic inversion of the tachocline thickness $h \leq 0.05 R_\odot$ (Corbard et al. 1999). With the latitudinal dependence of the angular velocity at the base of the convection zone deduced from Thompson et al. (1996), $\Omega_{bcz}/2\pi = 456 - 72x^2 - 42x^4$ nHz (with $x = \cos\theta$), we have reestimated the coefficient $Q_4 = -1.707 \times 10^{-2}$, as well as the ratio between the rotation in the deep radiative zone and the equatorial rate $\Omega/\Omega_0 = 0.9104$. The prediction by Gough & Sekii (1997) for the latter, who consider instead the magnetic stresses, is ~ 0.96; presently, the seismic observations suggest a rotational ratio of 0.94 ± 0.01 (Corbard et al. 1999), which is intermediate between these two theoretical estimates.

An analysis of the dependence of our d and D_T with the global and differential rotation rates yields

$$D_T \propto \Omega \nu_H^{1/2} (\hat{\Omega}/\Omega)^2, \qquad d \propto \Omega^{1/2}/\nu_H^{1/4}. \tag{2}$$

Assuming that the turbulent viscosity is proportional to the differential rotation (i.e. $\nu_H \propto \hat{\Omega}$) and introducing the dependence of the differential rotation on

rotation observed by Donahue, Saar & Baliunas (1996) ($\hat{\Omega} \propto \Omega^{0.7\pm0.1}$), we finally obtain for $D_T(\Omega)$ and $d(\Omega)$ that

$$D_T \propto \Omega^{0.75\pm0.25}, \qquad d \propto \Omega^{(1.3\mp0.1)/4}. \tag{3}$$

We conclude that the tachocline mixing was stronger in the past both because that layer was thicker and because the diffusivity was larger. We render the mixing in the tachocline time dependent, through $D_T(\Omega(t))$ and $d(\Omega(t))$, by using the spin-down law $\Omega \propto t^{-1/2}$ which was deduced by Skumanich (1972) from the rotation rate of stellar clusters of different ages.

4. Results

Starting from the reference model of Brun, Turck-Chièze & Morel (1998) built with the CESAM code (Morel 1997), we introduce for this study the nuclear reaction ^{7}Li(p,α)^{4}He proposed by Engstler et al. (1992) and the coefficient D_T (equation 1) in the diffusion equation of chemical species, and we follow the time evolution of the solar model from the pre-main sequence until 4.6 Gyr. The results are shown in the Table 1 and Figures 2-5 (see Brun, Turck-Chièze & Zahn 1999 for a more detailed discussion). We use a tachocline thickness h of 0.05 or 0.025 $R_\odot$, N of 100 or 25 μHz and Ω of 0.415 μHz. Our standard model (Brun, Turck-Chièze & Morel 1998) has a surface abundance for helium of 0.2427 in mass, corresponding to an ^{4}He difffusion of 10.8%. This value of Y_s is a bit too low if we compare with the Basu & Antia (1995) value for the OPAL equation of state (Rogers, Swenson & Iglesias 1996), $Y_s = 0.249 \pm 0.003$.

Table 1. Surface abundance variation of ^{3}He/^{4}He during the last 3 Gyr, surface abundances of ^{4}He and heavy elements Z, and abundance ratio initial/surface for ^{7}Li and ^{9}Be from observations and for solar models at the solar age (subscripts t for time dependent models, tz for time dependent model with $Z_0 = Z_0^{ref} = 0.01959$ and to for time dependent model with Caughlan & Fowler (1988) ^{7}Li(p,α)^{4}He nuclear reaction rate)

	Obs	Ref	D_{TA}	D_{TB}	D_{TAt}	D_{TBt}	D_{TBtz}	D_{TBto}	D_{TCt}
$d\ (r/R_\odot)$	≤ 0.1	-	0.1	0.1	0.1	0.1	0.1	0.1	0.05
N (μHz)	-	-	100	25	100	25	25	25	25
$(^3\mathrm{He}/^4\mathrm{He})_s$	max 10%	2.28%	2.14%	2.01%	2.11%	2.0%	2.02%	2.0%	2.07%
$^4\mathrm{He}_s$	0.249±0.003	0.2427	0.2452	0.2473	0.2455	0.2477	0.2509	0.2477	0.2464
$(\mathrm{Z/X})_s$	0.0245±0.002	0.0245	0.0245	0.0245	0.0245	0.0245	0.0255	0.0245	0.0245
$^7\mathrm{Li}_0/^7\mathrm{Li}_s$	$\sim$100	$\sim$6	$\sim$8	$\sim$22	$\sim$12	$\sim$91	$\sim$134	$\sim$64	$\sim$89
$^9\mathrm{Be}_0/^9\mathrm{Be}_s$	1.10±0.03	1.115	1.093	1.086	1.093	1.118	1.125	1.118	1.092

When introducing our diffusive coefficient, we mix helium back into the convection zone, inhibiting the microscopic diffusion up to 25% and leading to a photospheric $^4\mathrm{He}_s = 0.2473$ (cf. Table 1, models D_{TA} and D_{TB}). As expected, the composition profile is smoother and flattens over the distance h below the convective zone (see Figure 3).

The two light chemical elements ^{7}Li and ^{9}Be are really sensitive to mixing processes occuring in the stars because their nuclear burning temperatures are very low (resp. 2.5 10^6 and 3.2 10^6 K) (Baglin & Lebreton 1990). The new observational constraints can only be satisfied if those chemical species are mixed in a rather thin layer below the convective zone, which is the case for the tachocline mixing considered here. We see in Table 1 that the mixing increases the lithium

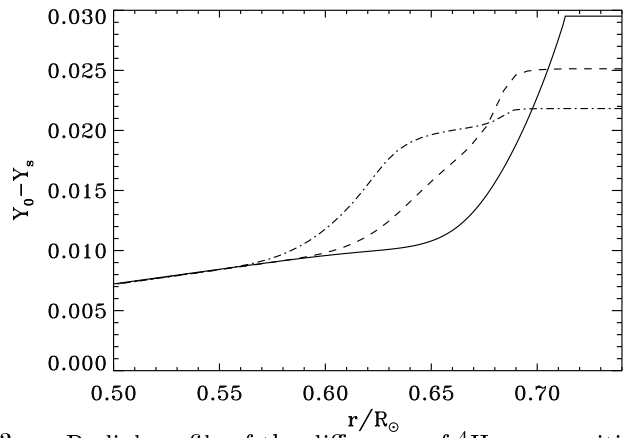

Figure 3. Radial profile of the difference of ^{4}He composition between the initial and present values for the reference solar model including only microscopic diffusion (solid line) and solar models where we add a macroscopic mixing due to the presence of the tachocline: coefficient D_{TA} (dash) and D_{TB} (dash dot) (see Table 1 for the model characteristics).

depletion by a factor up to ~ 4, without burning ^{9}Be, which agrees with the revised determination by Balachandran & Bell (1998). Nevertheless, such an increase in the ^{7}Li depletion is still insufficient to reproduce the photospheric lithium depletion (Cayrel 1998 and references therein). For the models with an added index t (as D_{TBt}), we calculate the evolution with the time-dependent diffusivity of equation (3).

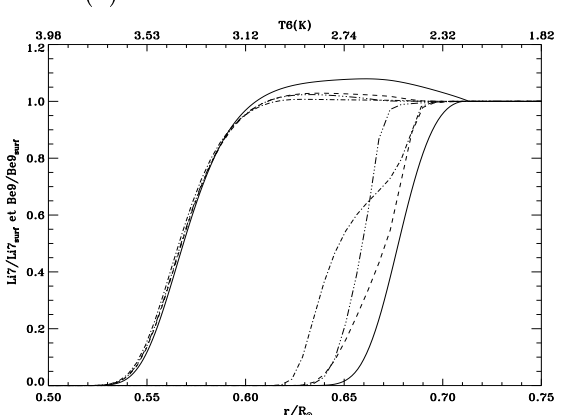

Figure 4. ^{9}Be and ^{7}Li radial profile (approximately superimposed the temperature scale) for several models. Respectively: reference (solid line), coefficient D_{TAt} (dash), coefficient D_{TBt} (dash dot) and coefficient D_{TCt} (dash three dots).

In Table 1 we give the initial over present ratio of ^{7}Li and ^{9}Be and show in Figure 4 the radial profile of ^{7}Li and ^{9}Be normalized to the surface abundance. We clearly see that the mixing process modifies the distribution of lithium but not that of beryllium (exception being the flat plateau for the mixed models in comparison with the "pure" diffusive one). With the coefficients D_{TB} more ^{7}Li is burned than with D_{TA}, and we also see that the time dependence (model with index t) improves the ^{4}He surface abundance as well as the ^{7}Li depletion, where

a value of ~ 100 is obtained without destroying ^{9}Be or increasing too much the ^{3}He/^{4}He surface ratio over the past 3 Gyr, as deduced by Geiss & Gloeckler (1998) from meteorites and solar wind abundance measurements (see Table 1).

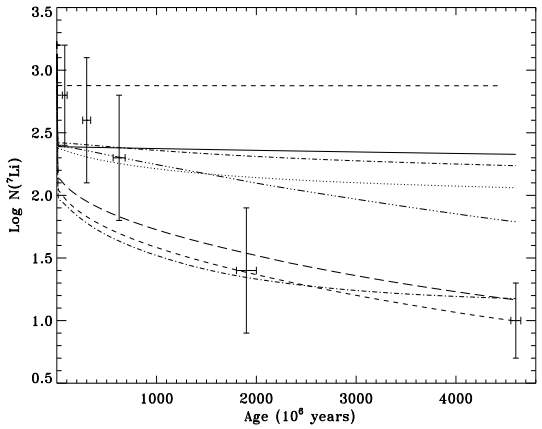

Figure 5. Time dependent depletion of ^{7}Li for several solar models: no microscopic diffusion (- - -), with microscopic diffusion (solid line) and with mixing in the tachocline thickness: coefficient D_{TA} (-.-), D_{TB} (-...-), then with time dependent mixing D_{TAt} (...) and D_{TBt} (long dash), D_{TCt} (thick -.-) and D_{TBtz} ($Z_0 = Z_0^{ref} = 0.01959$) (thick - -). We superimposed on the theoretical curves the open cluster observations (adapted from Vauclair & Richard 1998 and cluster age uncertainties deduced from Lebreton et al. 1997)

In Figure 5 we show the lithium depletion occurring during the Sun's evolution for the different models presented, plus a model without any diffusion. Clearly, only the diffusive models including mixing in the tachocline yield a substantial depletion during main sequence evolution, in agreement with the observations (superimposed with their inherent dispersion on the theoretical curves). Note that the strong time dependent mixing with $N = 25$ (models D_{TBt}, D_{TCt} and D_{TBtz}) presents a reasonable value of the solar ^{7}Li depletion (~ 100). We also indicate in Table 1 the sensitivity of the ^{7}Li depletion to the nuclear reaction rate ^{7}Li(p,α)^{4}He. The model D_{TBto}, with a depletion reduced by $\sim 40\%$ corresponds to the previous recommendation of Caughlan & Fowler (1988).

However the lithium depletion during the PMS is probably overestimated due to the crude spin-down law we have adopted. A more detailed analysis of these phases is under studies, including metallicity effects and more appropriate angular momentum evolution during this phase.

When comparing the sound speed profile with the helioseismic inversion (Figure 2), we see that the prominent bump is practically erased by this process. The model D_{TBtz} with an initial heavy element composition $Z_0 = Z_0^{ref} = 0.01959$ seems even in better agreement with the helioseismic data, keeping a Z/X surface abundance within the observational error bars (cf. Table 1).

Our results show the interest to follow together the photospheric abundance of the four elements ^{3}He, ^{4}He, ^{7}Li, ^{9}Be, and to examine their sensitivity to the microscopic as well as the macroscopic processes. This study encourages the introduction of macroscopic processes in stellar evolution models in the thin tachocline layer below the convective zone (see also Vauclair & Richard 1998).

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